Math 250 Fall 2008 Practice Problems for Exam 1 (Woodward); Actual exam will have 6 instead of 9 problems

PROBLEM 1 Find (a) a row echelon form and (b) the reduced row echelon form and (c) an LU factorization for the matrix

$$\begin{bmatrix} 1 & -1 & 0 & -8 & -3 \\ -2 & 1 & 0 & 9 & 5 \\ 3 & -3 & 0 & -2 & -11 \end{bmatrix}$$

(d) Using your answer to (b), find all solutions to the system whose augmented matrix is A.

PROBLEM 2 True or False? Justify your answer.

(a) If A and B are matrices such that $AB = I$ then A and B are invertible.

(b) If the number of columns of A is more than the number of rows, then $Ax = b$ has infinite solutions.

(c) The nullity of a matrix is the number of free variables.

(d) The inverse of an invertible upper triangular matrix is upper triangular.

(e) The identity matrix I is invertible.

(f) If $v_1, \ldots, v_m$ are vectors in R^n, and $m \geq n$, then these vectors span R^n.

PROBLEM 3 Compute (a) A^2 (b) A^{-1} and (c) A^T for $A = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 1 & 2 \\ 3 & 0 & 0 \end{bmatrix}$.
You might want to check your answer by multiplying A by A^{-1}.

PROBLEM 4

Find the determinant for

$$A = \begin{bmatrix} 2 & 0 & -5 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & 0 & 5 \\ 2 & 0 & 6 & -2 \end{bmatrix}$$

by any method.

PROBLEM 5 Find a set of vectors that is as small as possible that has the same span as $\{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\}$.

PROBLEM 6 Find all solutions to the system

$$\begin{aligned} x_1 - x_2 + -8x_4 &= -3 \\ -2x_1 + x_2 + 9x_4 &= 5 \\ 3x_1 - 3x_3 - 2x_4 &= -11 \end{aligned}.$$

PROBLEM 7 Prove that if v_1 and v_2 are linearly independent, then v_1 and $v_1 + v_2$ are linearly independent.

Prove that if a matrix A is invertible, then $A^T A$ is also invertible.

PROBLEM 8 True or False? Justify your answer.

(a) If A is invertible then A^T is invertible. (b) If A is invertible then $ref(A)$ is invertible. (c) If the number of columns of A is equal to the number of rows, then $Ax = b$ always has a unique solution. (d) A matrix whose diagonal entries are all zero is not invertible. (e) The identity matrix I is invertible. (f) The inverse of a symmetric matrix is symmetric. (g) If A and B are symmetric, then AB is symmetric. (h) If the number of columns of A is greater than the number of rows, then $Ax = b$ cannot have a unique solution. (i) If A and B are symmetric matrices, then $A + B$ is also symmetric. (j) If $v_1, \ldots, v_m$ are vectors in R^n, and $m = n$, then these vectors span R^n.

PROBLEM 9

Find the unique quadratic (degree 2) polynomial passing through the points $(0, 1), (1, 2), (-1, 3)$