Math 250 Practice Final Exam Fall 2008

PROBLEM 1 (a) Find a basis for the space V of solutions to the equations $a+b+c+d+e=0,\ c+d=0$ in R^5. (b) Make the basis orthonormal, using Gram-Schmidt. (c) Find the orthogonal projection of the vector $v=[0,0,0,0,1]$ onto V. (d) Find the distance of v from V. (e) Find the matrix for the orthogonal projection onto V.

PROBLEM 2 Let $A=\begin{bmatrix}1&2&3&0\\4&5&6&0\\7&0&0&0\\0&0&0&2\end{bmatrix}$. (a) Find the determinant of A, by expanding along the third column. (b) Find the inverse of A, by row reduction. (c) Find the square A^2 of A.

PROBLEM 3 (a) Let $f(t)$ be a function of the form $f(t)=c_0+c_1|t|+c_2t$. Write the system of equations $f(-1)=0,\ f(0)=0,\ f(1)=2$ in matrix form $Ax=b$. (b) Using row reduction, find all solutions to this system (i.e. function satisfying these conditions.) (c) Find the LU factorization for A. (d) Write a basis for the column-space, row-space, and null-space of A. (e) Write the dimensions of the null-space, row-space, and column-space for A. (f) Using least-squares approximation find all the functions of the form $f(t)=c_0+c_1|t|$. which are best fits for the data points $(-1,0),(0,0),(0,2)$. (g) Graph your answer(s) from part (f).

PROBLEM 4 True or false: Justify your answer in one or two sentences.

(a) If A has the same number of rows as columns, then any linear system of equations $Ax=b$ has a unique solution.
(b) If $\det(A)=0$, then $Ax=0$ has infinite solutions.
(c) Suppose A is a square matrix with row vectors v_1,v_2,v_3. If $\det(A)=2$, then the determinant of the matrix A' with row vectors $v_1+v_2+v_3, v_2+v_3, v_3$ is also 2.
(d) The vectors $[-1\ 1\ 0],[1\ 0\ -1],[0\ 1\ -1]$ are independent.
(e) If P is the matrix for orthogonal projection onto a subspace V, then $P^3=P$.
(f) If A is diagonalizable, then so is A^T.
(g) The rows of an orthogonal matrix form an orthonormal basis.
(h) If A is not invertible, then 0 is an eigenvalue.
(i) If $z=\sqrt{3}/2+i/2$ then $z^{30}=1$.
(j) The area of the triangle with vertices $[1\ 1],[-2\ 0]$ and $[-2\ -1]$ is $5/2$.

PROBLEM 5

(a) Find the (real and complex) eigenvalues for the matrix $A=\frac{1}{3}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$.
(b) Find the diagonalization of A.
(c) What is A^5? (Hint: very little computation necessary.)

PROBLEM 5' Same for

$$A=\begin{bmatrix}0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\\1&0&0&0&0\end{bmatrix}$$

PROBLEM 6 A party is going on in two rooms. Each hour, 70 percent of the people in room 1 move to room 2, and 80 percent of the people in room 2 move to room 1.
(a) Write the system in matrix form, i.e. find a matrix A such that $Ax(t+1)=x(t)$, where $x(t)$ is the distribution vector. (b) Find the percent of customers with each company, in the long run. (c) Suppose that initially, 100 people are in room 1.

PROBLEM 7 Let $q(x_1,x_2)=2x_1^2-4x_1x_2-x_2^2$. Find coordinates y_1,y_2 and numbers λ_1,λ_2 such that $q(y_1,y_2)=\lambda_1y_1^2+\lambda_2y_2^2$.

PROBLEM 8 (a) Give an example of a matrix that is not diagonalizable. (b) Give an example of a set of non-zero vectors that is not independent. (c) Give an example of an orthogonal matrix that is not the identity. (d) Give an example of a subset of R^2 that is closed under scalar multiplication but not under vector addition. (And so is not a subspace.)

PROBLEM 9 (i) Show that if A^2 is invertible, then so is A. (ii) Show that if v_1,v_2 are independent vectors, then so are v_1,v_1-v_2. (Hint: suppose there is a dependence relation on v_1,v_1+v_2.) (iii) Show that if 1 is an eigenvalue for A, then 1 is an eigenvalue for A^2. (iv) Prove that if A is diagonalizable, then so is A^{-1}.