Practice Midterm 2 Fall 2002

PROBLEM 1

Find a basis for the column-space, null-space, and row-space of the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 4 & 2 \\ 1 & -1 & 3 & 0 \end{bmatrix}.$$

(b) What are the rank and nullity of A? (c) Write down a dependence relation on the columns of A. (d) Find an LU factorization of A.

PROBLEM 2 (a) Find the determinant for $A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ by expanding along the second row.

(b) Find the determinant of A, using properties of the determinant under row-reduction.

PROBLEM 3

True or false: Justify your answer in one or two sentences.

(a) If an invertible square matrix A is diagonalizable, then so is A^{-1}.

(b) Suppose A is a square matrix with row vectors v_1, v_2, v_3. If $det(A) = 2$, then the determinant of the matrix A' with row vectors $v_1 + v_3, v_2, v_1 + v_2 + v_3$ is also 2.

(c) The dimension of the column space of A is the same as the dimension of the row-space.

(d) If A is a 7×3 matrix whose rank is 3, then the rows of A are linearly independent.

(e) If $Q = Q^{-1}$, then $det(Q) = \pm 1$.

(f) The row space is the orthogonal complement of the nullspace.

(g) If A has rank 0, then A is the zero matrix.

(h) For any square matrix A, $det(-A) = -det(A)$.

PROBLEM 4 Find the eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Find the diagonalization of A, if it exists.

PROBLEM 5

Three companies are competing for customers. Each year, company A loses all its customers to company B¡ company C loses 80 percent of its customers to company B, while company B each year loses 70 percent of its customers to company C. (Clearly neither of the company's produce a very high quality product!)

(a) Write down the state vector and time evolution matrix for this system. That is, represent the system in the form $x(t+1) = Ax(t)$, for some matrix A.

(b) Find the diagonalization of the matrix A.

(c) Suppose that initially, 100 customers are with company A and none with company B,C. Find a formula for the number of customers with company C at time t.

(d) How many customers does C have, for t very large.

(e) Show the evolution of the system on the graph with axes A,B.

PROBLEM 6

(a) Show that λ is an eigenvalue of A, if and only if $\det(A - \lambda I) = 0$.

(b)Show that if v_1, v_2 are eigenvectors of a matrix A with different eigenvalues λ_1, λ_2 then $\{v_1, v_2\}$ is a ilnearly independent set of vectors.