

Math 250 Practice Final Exam Spring 2002

PROBLEM 1 Find all solutions of the following linear system of equations.

$$2x + y - 3z = -2$$

$$6x + 3y - 8z = -5$$

$$2x + y + 5z = 6$$

PROBLEM 2

(a) Find the cofactor matrix for $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 0 \end{bmatrix}$. (b) Find the determinant of

A , by expanding along the third column.

(c) Find the inverse of A , using parts (a),(b).

(d) Find the inverse of A , by row reduction.

PROBLEM 3

(a) Let $f(t)$ be a function of the form $f(t) = c_0 + c_1|t| + c_2|t|^2$. Write the system of equations $f(-1) = 1$, $f(0) = 0$, $f(1) = 1$ in matrix form $Ax = b$.

(b) Using row reduction, find all solutions to this system (i.e. function satisfying these conditions.)

(c) Find the LU factorization for A .

(d) Write a basis for the column-space, row-space, and null-space of A .

(e) Write the dimensions of the null-space, row-space, and column-space for A .

(f) Find all the functions of the form $f(t) = c_0 + c_1|t| + c_2|t|^2$ which are best fits for the data points $(-1, 1)$, $(0, 0)$, $(1, 0)$.

PROBLEM 4

True or false: Justify your answer in one or two sentences.

(a) If A has the same number of rows as columns, then any linear system of equations $Ax = b$ has a unique solution.

(b) Any projection matrix P is diagonalizable.

(c) If A is diagonalizable, then so is A^2 .

(d) The rows of an orthogonal matrix form an orthonormal basis.

(e) If $AA^T = 0$ for some matrix A , then $A = 0$.

(f) If A is an $n \times n$ matrix with $\text{rank}(A) = n$, then A is invertible.

(g) If a square matrix A is diagonalizable, then so is $2A$.

(h) If A has rank 0, then A is the zero matrix.

(i) Suppose A is a square matrix with row vectors v_1, v_2, v_3 . If $\det(A) = 2$, then the determinant of the matrix A' with row vectors $v_1 + v_2 + v_3, v_2 + v_3, v_3$ is also 2.

PROBLEM 5

(a) Find the (real and complex) eigenvalues for the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(b) Find the diagonalization of A .

(c) What is A^5 ? (Hint: no computation necessary.)

(c) Find the eigenvalues for the matrix $I - A$. (Hint: no computation necessary.)

PROBLEM 6

- (a) Find the (real and complex) eigenvalues for the matrix $A = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.
- (b) Find the diagonalization of A .
- (c) What is A^5 ? (Hint: no computation necessary.)
- (d) Find the eigenvalues for the matrix $I - A$. (Hint: no computation necessary.)

PROBLEM 7

Determine whether the following maps T are linear transformations. If T is linear, find the matrix A such that $T[x] = Ax$. If T is not linear, explain why.

- (a) $T[x_1, x_2, x_3] = [x_1 - x_2 + 3x_3, x_1 + x_2 + x_3, x_1]$.
- (b) $T[x_1, x_2] = [x_1, 1/x_2]$.
- (c) $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is rotation counterclockwise by 210 degrees, about the point $(0, 0)$.
- (d) $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is orthogonal projection onto the $x - y$ plane.

PROBLEM 8

Two companies are competing for customers. Each year, company A loses 40 percent of its customers to company B, while company B each year loses 30 percent of its customers to company A.

- (a) Write down the state vector and time evolution matrix for this system.
- (b) Find the percent of customers with each company, in the long run.
- (c) Suppose that initially, 100 customers are with company A. Show the evolution of the system on the graph with axes A,B.

PROBLEM 9

- (a) Find an orthonormal basis for the subspace V of \mathbb{R}^4 defined by the equation $2x - y - z - w = 0$.
- (b) Find the projection of the vector $v = [0, 0, 0, 1]$ onto V .
- (c) Find the distance of v from V .
- (d) Find the projection of v onto the orthogonal complement V^\perp .
- (e) Find the matrix for the orthogonal projection of V , in terms of the standard basis for \mathbb{R}^4 .
- (f) Find the matrix for the projection onto V , in terms of the basis given in part (a).

PROBLEM 10

Let $q(x_1, x_2) = 2x_1^2 - 4x_1x_2 - x_2^2$. Find coordinates y_1, y_2 and numbers a, b such that $q(y_1, y_2) = ay_1^2 + by_2^2$.

PROBLEM 11

Let A be a square matrix. Show that A and A^T have the same eigenvalues.

PROBLEM 12

Suppose that u_1, \dots, u_n is an orthonormal basis for \mathbb{R}^n , and let $v = c_1u_1 + \dots + c_nu_n$. Show that $\|v\| = \sqrt{c_1^2 + \dots + c_n^2}$.