Practice Midterm 2 Spring 2002

**PROBLEM 1**

Find a basis for the column-space, null-space, and row-space of the matrix

$$A = \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & -2 & 4 & 2 \\ 1 & -1 & 3 & 0 \end{bmatrix}.$$

What are the rank and nullity of $A$? Write down a dependence relation on the columns of $A$.

**PROBLEM 2**

(a) Find an orthonormal basis for the subspace $V$ that is the span of the vectors [1 1 1] and [−1 0 1].

(b) Find the matrix $P$ for orthogonal projection onto $V$.

(c) Find the projection of the vector [1 0 2] onto $V$.

(d) Find the closest point to [1 0 2] in $V^{\perp}$.

**PROBLEM 3**

(a) Using the least squares method, find the function of the form $f(x) = a + bx^3$ that best fits the data points $(-1,-1), (0,0), (1,3)$.

(b) Compute the values of the function $f(x)$ that you found in (a) at $x = -1, x = 0$ and $x = 1$.

(c)Draw a rough sketch of the function $f(x)$ that you found in (a).

**PROBLEM 4**

True or false: Justify your answer in one or two sentences.

(a) If $Q$ is an orthogonal matrix, then so is $Q^T$.

(b) If $P$ is a projection matrix, then $P^3 = P$.

(c) The dimension of the column space of $A$ is the same as the dimension of the row-space.

(d) If $Q$ is orthogonal, then $\det(Q) = \pm 1$.

(e) If $V$ is a subspace of $R^n$ of dimension $k$, then the orthogonal complement also has dimension $k$.

(f) The row space is the orthogonal complement of the nullspace.

(g) If $P$ is a projection matrix onto $V$, then $I - P$ is the projection matrix onto $V^{\perp}$.

(i) If $A$ has rank 0, then $A$ is the zero matrix.

**PROBLEM 5** Find the eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

**PROBLEM 6**

Find all the functions of the form $f(t) = c_0 + c_1 \cos(t) + c_2 \cos(2t)$ which are best fits for the data points

**PROBLEM 7**

(a) Find a basis for the plane $V$ in $R^4$ *perpendicular* to [2 1 0 1] and [0 1 1 0].

(b) Make the basis you found in part (a) into an orthonormal basis, using Gram-Schmidt.

(c) Find the matrix for projection onto $V$.

(d) Find the projection of $b = [0\ -1\ 0\ 1]$ onto $V$.

(e) Find the distance from $b$ to $Pb$.

(f) Find the angle between $b$ and $Pb$. Your answer may involve an inverse trig function.

**PROBLEM 8**