Math 250 Spring 2002 Practice Exam 1 (Woodward)

PROBLEM 1 (a) Find the distance between the points $(1,0,1)$ and $(0,1,2)$ in R^3. (b) Find a UNIT vector perpendicular to $v = [-1\ 1\ 2]$ in R^3. (c) Find the angle between the vectors $[2\ 2\ 1]$ and $[2\ -1\ 2]$.

PROBLEM 2 Find all solutions to the system

$$\begin{aligned} x_1 - x_2 + x_3 + 2x_4 &= 3 \\ 2x_1 - 2x_2 + 4x_3 + 2x_4 &= 6 \\ x_1 - x_2 + 3x_3 &= 3 \end{aligned}.$$

What are the solutions to the corresponding homogeneous system?

PROBLEM 3 True or False? Justify your answer.

(a) If A is invertible then A^T is invertible. (b) If the number of columns of A is equal to the number of rows, then $Ax = b$ always has a unique solution. (c) The set of vectors $[x\ y\ z]$ with $y = 2x + 1$, $z = 2x - 1$ is a subspace of R^3. (d) The identity matrix I is invertible. (e) If A and B are symmetric, then AB is symmetric. (f) If the number of columns of A is greater than the number of rows, then $Ax = b$ cannot have a unique solution. (g) If A and B are symmetric matrices, then $A + B$ is also symmetric. (h) For any matrix A, $\det(-A) = -\det(A)$. (i) For any matrix A, $\det(A^{-1}) = \det(A)$. True or false continued. (j) $A = [v_1|v_2|v_3]$ is the matrix with columns v_1, v_2, v_3. If $\det(A) = 2$, then the determinant of the matrix $A' = [v_1 - v_2|v_2 - v_3|v_3 - v_1]$ is also 2.

PROBLEM 4

Let $A = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 1 & 0 \\ 1/3 & 0 & 0 \end{bmatrix}$. Compute (a) A^2 (b) A^{-1} and (c) A^T.

PROBLEM 5

Find the inverse of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

by two different methods: (1) cofactors (2) elimination. You might want to check your answer by multiplying A by A^{-1}.

(b) Find the transpose of A.

PROBLEM 6

Find the determinant of the matrix $A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ 1 & 3 & 9 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ using two different methods: (1) row or column operations and (2) cofactor expansion.

PROBLEM 7 Find the inverse of

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

using cofactors.

PROBLEM 8

Find the unique quadratic (degree 2) polynomial passing through the points $(0,1), (1,2), (-1,3)$.

PROBLEM 9

Prove that if the equation $Ax = b$ has more than one solution, then it has infinitely many solutions.

PROBLEM 10

Determine whether the following maps T are linear transformations. If T is linear, find the matrix A such that $T[x] = Ax$. If T is not linear, explain why. (a) $T[x_1, x_2, x_3] = [x_1 - x_2 + 3x_3, x_1 + x_2 + x_3, x_1]$. (b) $T[x_1, x_2] = [x_1, 1/x_2]$. (c) $T:\ R^2 \to R^2$ is rotation counterclockwise by 210 degrees, about the point $(0,0)$.