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Solve problems 1–14 at the end of VII§5.

\* Let  $F$  be a nonconstant fractional linear transformation which maps the circle  $C(a, r)$  onto the circle  $C(A, R)$ . Find a simple proof that if  $F$  preserves orientation (i.e. if  $z_1, z_2, z_3$  are close to each other and  $z_2$  lies between  $z_1$  and  $z_3$  then  $F(z_2)$  lies between  $F(z_1)$  and  $F(z_3)$ ), then the disk  $D(a, r)$  is mapped (one to one) to the disk  $D(A, R)$ , and otherwise  $D(a, r)$  is mapped on the *exterior* of  $D(A, R)$ . Formulate and prove a similar statement for fractional linear transformation of lines into circles and lines into lines.