1. For $z \in \mathbb{C}$ define $\exp(z) = \sum_{k=0}^{\infty} z^k/k!$. Using the properties of the exponential function of a real variable show that

$$\lim_{n\to\infty}\left(1+\frac{z}{n}\right)^n = \exp(z)$$

for all $z \in \mathbb{C}$.

2. [1] If $\omega = \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$ show that

$$1 + \omega^k + \omega^{2k} + \ldots + \omega^{(n-1)k} = 0$$

for any integer k which is not a multiple of n.

3. [1] Show that an analytic function cannot have constant absolute value without reducing to a constant.

4. Let $f(z) = \sum_{k=0}^{n} a_k z^k$ be a polynomial with complex coefficients and let $u(x,y) = \Re f(z)$ (as usual, $z = x+iy$). Show that $f(z) = 2u(z/2, -iz/2) + const.$

5. Assume g is a polynomial in the two real variables x and y with complex coefficients, $g = \sum_{0\le k,l\le n} a_{k,l}x^k y^l$ and let $u = \Re g, v = \Im g$. Show that u, v satisfy the C-R equations iff $g = \sum_{k=0}^{p} b_k(x+iy)^k$ for some p and complex constants $b_0, \ldots, b_p$.

[1] Ahlfors *Complex Analysis*, Third edition.