

Think & Derive

Chris Long

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Problem: In Dickens' novel "Great Expectations", Biddy is described as Mr. Wopsle's great-aunt's grand-daughter. Pip confesses that he's quite unable to work out the relationship between Biddy and Mr. Wopsle. What is it?

Solution: The common ancestors here are the great-aunt's parents (which are Biddy's and Mr. Wopsle's great-grandparents). The number of generations from Biddy to these common ancestors is up 3; from Mr. Wopsle to these common ancestors is up 3. Thus Biddy and Mr. Wopsle are 2nd cousins. \square

Problem: Estimate the relationship between you and a common grey squirrel. Since you have common ancestors, you're cousins; the puzzle is to estimate what kind of cousins you are. Assume that humans and squirrels had a common ancestor 100,000,000 years ago (as according to current theory), that the squirrel branch had a constant generational length of 6 months, and that the human branch had a generational length that increased linearly from 6 months to 25 years.

Solution: The squirrel branch is in the 200,000,001st generation since the bifurcation.

The human branch is more difficult. Let $T(k)$ be the time the k th generation starts (so $T(1) = 0$, $T(2) = 0.5$), and let $g(t)$ be the length of the generation starting at time t . Then

$$g(t) = \frac{24.5}{10^8} \cdot t + 0.5 = \alpha \cdot t + 0.5,$$

where $\alpha = 24.5/10^8$. This gives the recursive equation

$$T(k+1) = T(k) + g(T(k)) = (\alpha + 1)T(k) + 0.5.$$

We may write this in the form $U(k+1) = (\alpha + 1)U(k)$, where $U(k) = T(k) - \frac{1}{2\alpha}$. Then $U(k) = \frac{1}{2\alpha}(\alpha + 1)^{k-1}$, and so $T(k) = \frac{1}{2\alpha}(\alpha + 1)^{k-1} - \frac{1}{2\alpha}$. We want to now find the largest k such that $T(k) \leq 10^8$, so

$$\frac{1}{2\alpha}(\alpha + 1)^{k-1} - \frac{1}{2\alpha} \leq 10^8,$$

or

$$(\alpha + 1)^{k-1} \leq 2\alpha \cdot \left(10^8 + \frac{1}{2\alpha}\right) = 50.$$

Taking logs and solving, $k \leq 15967443.79$, and so $k = 15967443$. Thus, the human branch is in the 15,967,443rd generation since the bifurcation.

This makes you and the squirrel 15,967,442nd cousins, 184,032,558 times removed. \square