Differential Equations 244

Sample Exam 2

Wednesday, February 28, 2001
Robert Doran
TA: Chris Long

1. (15) Solve the inhomogeneous ODE:

$$y'' + 4y = \cos 2x + xe^x.$$

2. (14) Solve the inhomogeneous ODE:

$$y'' + y = \cot t.$$

3. (14) An object of **mass** 1 slug is attached to a spring with a spring constant 1 lb/ft. The mass-spring system is connected to a weightless piston moving in an oil bath. The oil has a viscosity constant of 2 lb-sec/ft. The mass is lowered 3 **inches** below the equilibrium position and given an initial velocity of 1 ft/sec **upwards**.

   (a) (5) Set up the initial value problem. Make sure you state the location of your origin and the sign convention you adopt.

   (b) (6) solve the IVP of part (a).

   (c) (4) Graph the displacement versus time. From your graph indicate how many times your object crosses teh equilibrium position.

4. (14) Solve the homogeneous ODE:

$$\frac{d^4y}{dt^4} + y = 0.$$

5. (15)

   (a) (13) Find the general solution as a power series centered at $t = 0$ for the ODE:

   $$y'' - 2ty' - 2y = 0.$$

   Make sure you display the recurrence relation and display three non-zero terms for each of the basic solutions that constitute the general solution.

   (b) (2) What is the radius of convergence of the power series solution that you have obtained in part (a)?

6. (14) Given the ODE:

   $$t^2(t^2 + 4)(t - 1)y'' + y' - ty = 0,$$

   (a) (3) What are the ordinary points and what are the singular points for this ODE?

   (b) (5) Further classify the singular points as regular singular points and irregular singular points.

   (c) (6) Is it possible to find a solution of the displayed ODE, as a power series of the form $\sum_{n=0}^{\infty} a_n(t+3)^n$? If your answer is yes, what would be the radius of convergence of the power series solution? If your answer is no, give reasons why it is not justified in searching for a power series solution of the indicated form. You **DO NOT** have to compute the coefficients a_n in any of the cases, yes or no!!!

7. (14) Solve the ODE:

   $$t^2y'' + 4ty' + 7y = 0.$$