Differential Equations 244
Sample Exam 1

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1. (13) Given the ODE:
$$\frac{dy}{dt} = -ty$$
   (a) (3) Find the equations to the isoclines.
   (b) (6) On the t, y plane using (a) sketch the associated direction field of the ODE.
   (c) (4) On your direction field diagram sketch the solution that satisfies the ODE and the initial condition $y(0) = 1$.

2. (13) Given the ODE:
$$(ye^{2ty} + t)dt + bte^{2ty} = 0$$
   (a) (6) Compute b so that the displayed ODE becomes an exact equation.
   (b) (7) Use this value of b that you have computed in part (a) and solve for the general solution of the ODE.

3. (12) A rain drop with **mass** 10 grams starts falling from a cloud. Assume that the air resistance it encounters has **magnitude** equal to $10|v|$ where v is the velocity in centimetres/sec.
   (a) (9) Find a formula for the velocity as a function of the time t. Assume that the value of the accelaration due to gravity **g** has the value 1000 cm/sec^2. **Make sure** you display the initial value problem.

(b) (3) Compute the limiting velocity v_l of the rain drop.

4. (14) Consider the ODE:

$$\frac{dp}{dt} = (p - 1)(p + 1).$$

(a) (2) Find the equilibrium solutions of the problem.

(b) (4) Draw a graph of $\frac{dp}{dt}$ versus p.

(c) (6) Using the graph of part (b) draw the solution curves that satisfy the initial conditions, $p(0) = 1.5$ **and** $p(0) = 0.5$.

(d) (2) What sort of stability/instability conclusions can you arrive at by your answers in part (c).

5. (13) In a certain chemical process, a tank whose capacity is 500 gallons contains 50 pounds of a chemical dissolved in 100 gallons of water. Now, chemical solution containing 1 pound/gallon of the chemical, is pumped into the tank at a rate of 3 gallons/min. The well-stirred mixture is pumped out of the tank at a rate of 3 gallons/min. Find a formula for the amount $m(t)$ of chemical in pounds at time t in the tank. **Make sure** you display the initial value problem.

6. (12) Solve the initial value problem:

$$y'' + 4y' + 5y = 0,\ y(0) = 1,\ y'(0) = 0.$$

7. (10) Given the two functions $f(t) = t^3$ and $g(t) = 3t^3 + t^2 - 1$. Determine by a computation that you make sure you display, whether the functions $f(t)$ and $g(t)$ are dependent or independent.

8. (13) Given that one solution of the displayed ODE is $y(t) = t$. Find the second linearly independent solution.

$$t^2y'' - t(t + 2)y' + (t + 2)y = 0,\ t > 0.$$