

Differential Equations 244

Exam 2 Review

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1. (3.6) Find the general solution of the given equation:

$$y'' - 2y' - 3y = 3e^{2t} + t + e^{-t}.$$

2. (3.6) Find the general solution of the given equation:

$$y'' + 4y' + 4y = 5e^{-2t} + 4.$$

3. (3.6) Find the general solution of the given equation:

$$y'' + y = 2\sin(t) + 2\cos(2t).$$

4. (3.7) Find the general solution of the given equation:

$$y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad t > 0.$$

5. (3.7) Find the general solution of the given equation:

$$t^2y'' - 2y = 3t^2 - 1, \quad t > 0; \quad y_1(t) = t^2, \quad y_2(t) = t^{-1}.$$

6. (3.8) A mass of 20 g stretches a spring 5 cm. Suppose that the mass is also attached to a viscous damper with a damping constant of 400 dyne-sec/cm. If the mass is pulled down an additional 2 cm and then released, find its position u at any time t . Plot u versus t . Determine the quasi frequency and the quasi period. Determine the ratio of the quasi period to the period of the corresponding undamped motion.

7. (3.8) A mass weighing 8 lb stretches a spring 1.5 in. The mass is also attached to a damper with coefficient γ . Determine the value of γ for which the system is critically damped.
8. (3.9) A mass that weighs 8 lb stretches a spring 6 in. The system is acted upon by an external force of $8 \sin(8t)$ lb. If the mass is pulled down 3 in. and then released, determine the position of the mass at any time. Determine the first four times at which the velocity of the mass is zero.
9. (4.1) Compute the Wronskian of $1, \cos(t), \sin(t)$. What can you conclude?
10. (4.1) Compute the Wronskian of $1, t, 3t + 1$. What can you conclude?
11. (4.2) Find the general solution of the given equation:

$$y''' - y'' - y' + y = 0.$$

12. (4.2) Find the general solution of the given equation:

$$y^{(4)} + 2y'' + y = 0.$$

13. (4.2) Find the general solution of the given equation:

$$y^{(6)} + y = 0.$$

14. (4.3) Find the general solution of the given equation:

$$y''' - y'' - y' + y = 2e^{-t} + 3.$$

15. (4.3) Find the general solution of the given equation:

$$y^{(4)} + y''' = \sin(2t).$$

16. (4.3) Find the general solution of the given equation:

$$y^{(6)} + y''' = t.$$

17. (5.1) Know the ratio and root tests, and how to compute the Taylor series of a given function.

18. (5.2-5.3) Solve the given equation by means of a power series about the given point x_0 :

$$y'' - y = 0, \quad x_0 = 0.$$

19. (5.2-5.3) Solve the given equation by means of a power series about the given point x_0 :

$$(1 - x)y'' + y = 0, \quad x_0 = 0.$$

20. (5.2-5.3) Determine a lower bound for the radius of convergence of series solutions about each given point x_0 :

$$xy'' + y = 0; \quad x_0 = 1, \quad x_0 = 2.$$

21. (5.4) Find all singular points and determine whether each one is regular or irregular:

$$x^2(1 - x^2)y'' + (2/x)y' + 4y = 0.$$

22. (5.5) Determine the general solution of the given equation that is valid in any interval not including the singular point:

$$x^2y'' - 3xy' + 4y = 0.$$

23. (5.5) Determine the general solution of the given equation that is valid in any interval not including the singular point:

$$2x^2y'' - 4xy' + 6y = 0.$$