Differential Equations 244

Exam 1 Review

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1. (2.1) Solve the initial value problem:

$$y' + 5y = t^2 + e^t,\ y(0) = 1.$$

2. (2.2) Solve the initial value problem:

$$y' = x(1 + y^2),\ y(0) = 1.$$

3. (2.3) A person borrows \$1000 at a rate of 7%, continuously compounded. They pay this loan back at a continuous rate of \$100 per year. How long will it take to repay the loan?

4. (2.3) A 100 gallon tank hold a well-mixed saline solution with a total of 10 pounds of salt. Fresh water comes in at a rate of 2 gallons/minute, and the well-mixed solution leaves at a rate of 2 gallons/minute. How long will it take before the amount of salt in the tank is less than 5 pounds?

5. (2.3) An anchor freely falls through the water, with the water offering a resistance of $2|v|$ newtons, where v is the velocity of the anchor. Assuming $v(0) = 0$, find a formula for $v(t)$ and also find the limiting velocity of the anchor (in meters/second).

6. (2.5) Consider the equation:

$$y' = -2 - 4y.$$

Sketch the graph, determine the critical points, and classify each critical point as either asymptotically stable, unstable, or semistable.

7. (2.5) Consider the equation:

$$dy/dt = y(1 - y^2).$$

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8. (2.6) Solve the initial value problem:

$$y' = (2x - y)/(x - 2y),\ y(1) = 3.$$

9. (2.6) Find an integrating factor and solve the given equation:

$$y' = e^{3x} + y - 1.$$

10. (3.1) Find the general solution of the given equation:

$$y'' + 2y' - 15y = 0.$$

11. (3.2) Compute the Wronskian of t^2, t^3. What can you conclude?

12. (3.2) Compute the Wronskian of e^x, e^{x+1}. What can you conclude?

13. (3.4) Solve the initial value problem:

$$y'' + 4y = 0,\ y(0) = 0,\ y'(0) = 1.$$

14. (3.5) Solve the initial value problem:

$$y'' - 6y' + 9y = 0,\ y(0) = 0,\ y'(0) = 2.$$

15. (3.5) Using reduction of order, find a second solution to the following differential equation:

$$t^2y'' + 2ty' - 2y = 0,\ t > 0,\ y_1(t) = t.$$