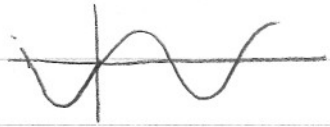
Parameterizing With Respect to Arc Length

- Why would you want to?

For a given curve there are many ways to write an equation describing it (ie. parameterizing it)

Compare $r_1(t) = \langle t, \sin t \rangle$ and $r_2(t) = \langle t^3, \sin(t^3) \rangle$

Both trace out curves that look like: 

But r_2 speeds up as $t \to \infty$, whereas r_1's speed fluctuates between 1 and 2 (calculate $|r_1'(t)|$ and $|r_2'(t)|$ to confirm this).

Sometimes a constant speed, say 1, is a desirable property.

Parameterizing by arc length does just that.

Why? because then distance travelled = "time"

- How do you parameterize by arc length? (measuring from $t=0$)
  1. Find arclength, s, in terms of t by $s = \int_0^t |r'(u)|\,du$.
  2. Solve for t in terms of s (hopefully this is not too hard)
  3. Substitute this into $r(t)$ to get r in terms of s.

Examples

1. See Example 2 in 13.3
2. Exercise 11 in 13.3: Parameterize $r(t) = \langle 3\sin t, 4t, 3\cos t \rangle$ by arc length.
   - Calculate: $r'(t) = \langle 3\cos t, 4, -3\sin t \rangle$

     $|r'(t)| = \sqrt{9\cos^2 t + 16 + 9\sin^2 t} = \sqrt{25} = 5$

     $s = \int_0^t |r'(u)|\,du = \int_0^t 5\,du = 5t.$
   - Solve: $t = s/5$
   - Substitute: $r(s/5) = \langle 3\sin(s/5), \frac{4s}{5}, 3\cos(s/5) \rangle$

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