Line Integrals (16.2)

Many students miss that there are, in fact, <u>two</u> kinds of line integrals: line integrals of a scalar function, and line integrals of a vector field.

Line Integrals of a Scalar Function.

Given: curve C, function $f(x,y)$

Asked to find: $\int_C f(x,y)\,ds$

Often C is given via parametric equations $x(t), y(t), a \le t \le b$.

Then $\int_C f(x,y)\,ds = \int_a^b f(x(t), y(t)) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$ $\boxed{3}$

Sometimes asked for $\int_C f(x,y)\,dx$ or $\int_C f(x,y)\,dy$.

$$\left.\begin{aligned} \int_C f(x,y)\,dx &= \int_a^b f(x(t), y(t))\, x'(t)\,dt \\ \int_C f(x,y)\,dy &= \int_a^b f(x(t), y(t))\, y'(t)\,dt \end{aligned}\right\} \boxed{7}$$

Of course, all of this can be done in three dimensions.

Line Integrals of a Vector Field*

Given: curve C, vector field $\vec{F}(x,y) = \langle P(x,y), Q(x,y) \rangle$

Usually given (or asked to find) function $\vec{r}(t) = \langle x(t), y(t) \rangle$ which describes C.

Asked to find $\int_C \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\,dt$ $\boxed{13}$

Notice that $\int_C \vec{F} \cdot d\vec{r} = \int_C P(x,y)\,dx + \int_C Q(x,y)\,dy$, which are line integrals over scalar functions

Again, all this can be done in three dimensions.

*16.3 and 16.4 deal with these kind when certain conditions are met.

Physical Interpretations

Figure 2 ↙ Example 1 ↙

Line integrals of a scalar field: Area of a sheet, Mass of a wire

Line integrals of a vector field: Work done by field.

Example 7

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