Change of Variables (15.9)

Reasons to change variables:

A. Simplify the integrand

eg. $\iint_R e^{(x+y)/x^2}\,dA \xrightarrow{\substack{u=x+y\\ v=x^2}} \iint_S e^{u/v}\left|\frac{\partial(x,y)}{\partial(u,v)}\right|dA.$

B. Simplify the region

eg. $\iint_R xy\,dA$ where R is elliptical disk $9x^2+4y^2\le 1$

Use the substitution $u=3x$, $v=2y$ to get

$\iint_S \left(\frac{u}{3}\right)\left(\frac{v}{2}\right)\left|\frac{\partial(x,y)}{\partial(u,v)}\right|dA$, for S the circular disk $u^2+v^2\le 1$

(From here you can use polar coordinates to finish)

Note

This ellipse to circle conversion is a common exam question

Procedure:

1. Decide on the substitution
   $u=?(x,y)$
   $v=?(x,y)$

2. Solve for x,y in terms of u and v.
   $x=?(u,v)$
   $y=?(u,v)$

(3 and 4 are independent from one another)

3. Compute Jacobian
   $\frac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$

4. Find new region of integration, S
   Use substitutions from 1 & 2 on the boundary curves of R.

5. Form the integral $\iint_S f(\overbrace{x(u,v),y(u,v)}^{\text{substitute in } u,v})\left|\frac{\partial(x,y)}{\partial(u,v)}\right|dA$.
   You'll need to replace dA with $du\,dv$ or $dv\,du$ and S with the appropriate bounds

6. Solve the integral

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