The following question from section 12.5 was asked in recitation section 16, but I did not have time to answer in class.

70. Find equations of the planes that are parallel to the plane $x+2y-2z=1$ and two units away from it.

First think about the geometry of the situation. When two planes are parallel, they're going to have the same normal vector. Recall we can read off the normal vector from the equation of a plane by looking at the coefficients of x, y and z. Thus the normal vector for any parallel plane is $\langle 1, 2, -2\rangle$. Now all that's left is to find the constant term.

There are two ways to go about this. One way is to use the formula from problem 69 which gives the distance between two planes. In this formula, you'll set $D = 2$ and $d_1 = -1$ (ask yourself: why negative?), and solve for d_2. That's the first way.

The second way is a little more complicated, but can reproduced without having that formula in front of you. We'll first need to find a point on the plane. Since the plane $x + 2y - 2z = 1$ is going to have to cross the x axis somewhere, we can set $y = 0$ and $z = 0$ to find exactly where. Solving for x you'll see that $x = 1$, so the plane crosses the x-axis at $(1, 0, 0)$. Thus $Q = (1, 0, 0)$ is a point on the plane. Once we have this, we travel two units in the direction of $\vec{n}$ to get a point on the plane two units "above" the plane $x+2y-2z = 1$ (above is in quotes because the normal vector actually points in the $-z$ direction). Since $\frac{\vec{n}}{|\vec{n}|}$ describes motion one unit in the direction of $\vec{n}$, we calculate:

$$\langle 1, 0, 0\rangle + 2\frac{\vec{n}}{|\vec{n}|} = \langle\frac{5}{3}, \frac{4}{3}, \frac{-4}{3}\rangle.$$

Thus $\left(\frac{5}{3}, \frac{4}{3}, \frac{-4}{3}\right)$ is a point on one of the planes. Now we get the equation $(x - 5/3) + 2(y - 4/3) - 2(z + 4/3) = 0$ which simplifies to be $x + 2y - 2z = 7$. Travelling two units in the direction opposite of $\vec{n}$ gets a point on the other plane. I'll leave it to you to do these calculations. You should get $x + 2y - 2z = -5$ as the equation for the second plane.