

Math 103 – Practice Final Exam
Summer Session II, 2008

Complete all questions. You may use scrap paper and a calculator, but no notes or textbook. You must show all work to qualify for full credit. **No work = no credit**. Please write neatly. If I can't read it, I can't grade it, and you can't get points.

1. (25 Pts) An election takes place amongst four candidates. After the ballots are collected, we get the following preference schedule:

	8	12	21
1 st	C	A	B
2 nd	D	C	A
3 rd	A	D	C
4 th	B	B	D

(a) Find the **winner** using the **Borda count method**.

(b) Find the **winner** using the **method of pairwise comparisons**.

(c) Does the election in (a) violate the **Majority Criterion**? Explain.

(d) Find the **ranking** of the candidates using the **extended plurality method**.

2. (25 Pts)

(a) Find the **Banzhaf power distribution** for the weighted voting system [16:12, 12, 4, 3] (Hint: you should get eight winning coalitions)

(b) Is any player a **dictator**? If so, who? Justify your answer.

(c) Is any player a **dummy**? If so, who? Justify your answer.

(d) Does any player have **veto power**? If so, who? Justify your answer.

For (e), (f), and (g), use the following scenario:

The Sunnydale High student advisory committee consists of three students and two faculty members. To pass a motion requires at least three votes, at least one of which must be a faculty member. Call the students S_1, S_2, S_3 , and the faculty F_1 and F_2

(e) If you were to write out all the sequential coalitions, how many would there be?

(f) In each sequential coalition below, circle the pivotal player.

$\langle F_1, S_2, S_1, S_3, F_2 \rangle$

$\langle F_2, F_1, S_3, S_2, S_1 \rangle$

$\langle F_1, S_1, F_2, S_2, S_3 \rangle$

$\langle S_1, S_3, S_2, F_2, F_1 \rangle$

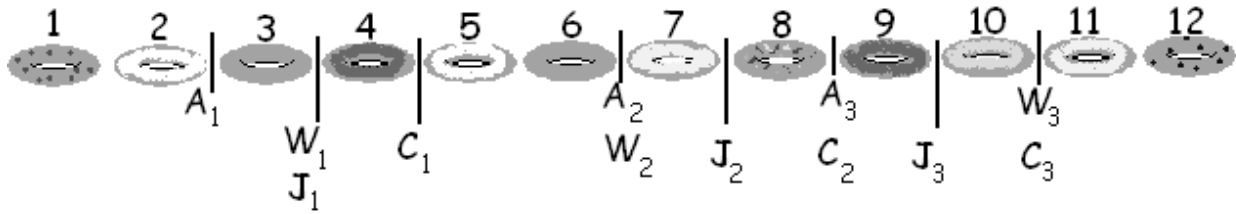
$\langle S_1, F_1, F_2, S_3, S_2 \rangle$

$\langle S_1, S_3, F_2, S_2, F_1 \rangle$

(g) If F_1 is pivotal 30 times, compute the Shapley-Shubik power distribution for F_1, F_2, S_1, S_2, S_3 .

3. (25 pts)

Will, Carlton, Ashley, and Jazz are splitting a box of a dozen donuts. To make sure no hilarity ensues, they decide to use the Method of Markers to keep everything fair.



(a) Which donuts does each person get?

Will:

Carlton:

Ashley:

Jazz:

(b) Which donuts are leftover?

(c) What do you think they should do with the leftovers? (There are no wrong answers here)

For (d)-(g), use the following scenario:

Rich Uncle Scrooge has bought Huey, Dewey, and Louie their own private island worth 300 million dollars. They decide to divide it amongst themselves using the Last-Diminisher Method. They order themselves randomly and will take turns in the order Huey, Dewey, then Louie.

(d) Huey marks off a portion of the island. How much should this portion be worth to Huey?

(e) Dewey goes next. He views Huey's portion as worth \$85 million. What should he do? After Dewey's turn, who is the claimant of the piece?

(f) Louie goes last. He views the claimed piece as worth \$110 million. What should he do? Who ends up with the first piece of the island? How much is it worth them?

(g) How should the remaining two players divide the rest of the island?

4. (25 pts) Eternia, Hyrule, Cimmeria, and Lilliput have decided to create a Fictional Alliance, with 174 seats to be divided among the four nations in proportion to their populations.

Nation	Eternia	Hyrule	Cimmeria	Lilliput
Population	221	204	177	148

(a) Calculate the standard divisor. How can this number be interpreted?

(b) Apportion the 174 seats using Hamilton Method.

Nation	Population				Hamilton Apportionment
Eternia	221				
Hyrule	204				
Cimmeria	177				
Lilliput	148				

(c) If the number of seats were increased to 175, we would get the apportionment:

Nation	Eternia	Hyrule	Cimmeria	Lilliput
Number of Seats	52	48	41	34

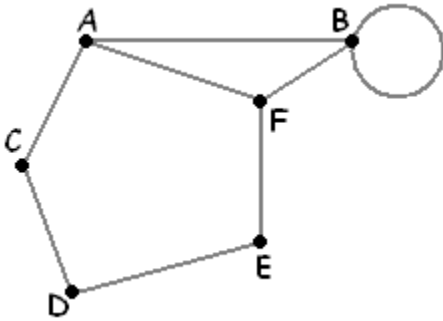
Does a paradox occur? If so, which paradox? Justify your answer.

(d) Use Adams's method to apportion the 174 seats (Timesaver: Either the number 4.25 or 4.35 will come in handy)

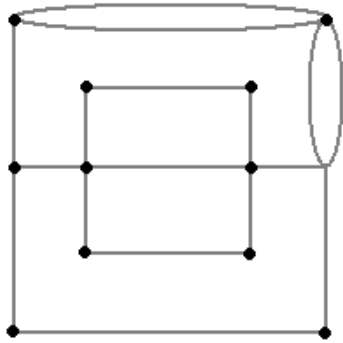
Nation	Population				Adams Apportionment
Eternia	221				
Hyrule	204				
Cimmeria	177				
Lilliput	148				

5. (25 pts)

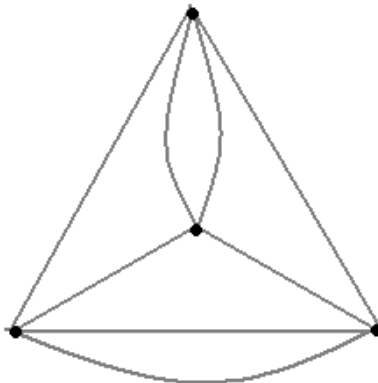
(a) Find a circuit of length 7 which starts and ends at A.



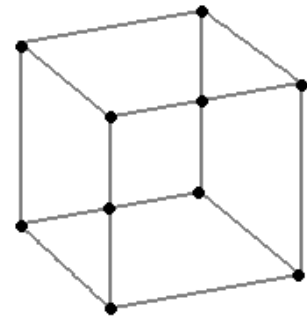
(b), (c), (d): In the graphs below, if there is an Euler path or circuit, mark it. If not, explain why not.



(b)

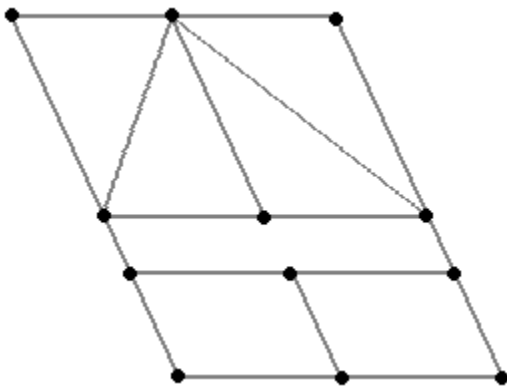


(c)



(d)

(e) Eulerize the graph below so that there is an Euler circuit. (Don't worry about making it optimal)

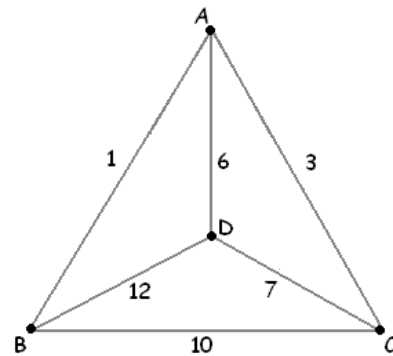


6. (25 pts)

(a) In the graph below, find a Hamilton path starting at A.

For (b), (c), and (d), use the graph to the right:

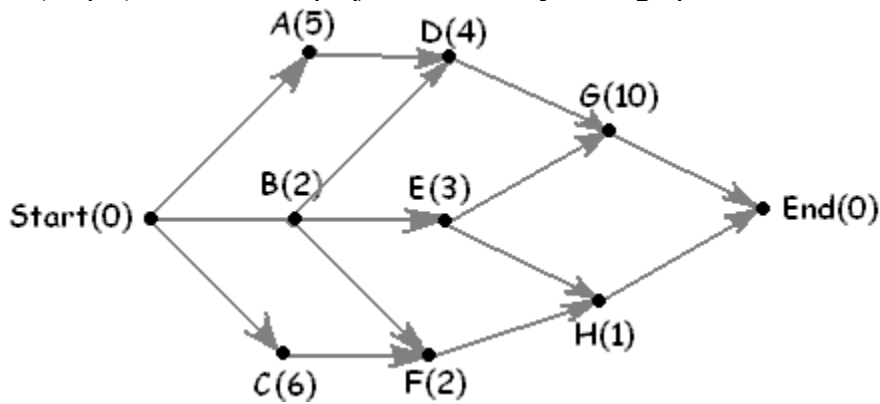
(b) Use the Brute-Force Algorithm to determine the minimal Hamilton circuit in the graph to the right. (Hint: There are 6 Hamilton circuits, but remember that some are the reverse of others)



(c) Use the Nearest-Neighbor Algorithm for the graph starting at vertex A to get a Hamilton circuit. Write the circuit starting and ending at B. What is its total cost? Is it the optimal circuit?

(d) Use the Cheapest-Link Algorithm to get a Hamilton circuit. Write the circuit starting and ending at B. What is its total cost? Is it the optimal circuit?

8. (25 pts) Consider the project outlined by the digraph below.



- (a) What is the decreasing-time priority list for this project?
- (b) Calculate the critical time for each task. Write your answers on the project digraph above.
- (c) What is the critical-path priority list for this project?
- (d) Use the Critical-Path Algorithm to determine a schedule for this project. Use the chart provided below (there is more horizontal space than necessary)

	0	1	2	5	10	15	20	25	30
P ₁									
P ₂									

- (e) What is the finishing time for the schedule in part (d)? Is this optimal? Why or why not?