Math 103 – Practice Exam I

Complete all questions. You may use scrap paper and a calculator, but no notes or textbook. You must show all work to qualify for full credit. **No work = no credit**. Please write neatly. If I can't read it, I can't grade it, and you can't get points.

1. (25 Pts) An election takes place amongst four candidates. After the ballots are collected, we get the following preference schedule:

	13	5	8	9
1st	A	B	C	C
2nd	B	A	B	A
3rd	D	D	A	B
4th	C	C	D	D

(a) Find the **winner** using the **method of pairwise comparisons**.

(b) Find the **winner** using the **plurality with elimination**.

(c) Is there a **majority candidate**? If so, who is it? If not, why not?

(d) Is there a **Condorcet candidate**? If so, who is it? If not, why not?

(e) Find the **ranking** of the candidates using the **extended Borda count**.

2. (20 Pts)
(a) Find the **Banzhaf power distribution** for the weighted voting system [12: 9, 5, 5, 1] (Hint: you should get six winning coalitions)

(b) Is any player a **dictator**? If so, who? Justify your answer.

(c) Is any player a **dummy**? If so, who? Justify your answer.

(d) Does any player have **veto power**? If so, who? Justify your answer.

For (e) and (f), use the following scenario: The city government of Coolsville is trying to implement the strong-mayor system. They set up a city council of three members and a mayor. If two of the three council members approve a motion, the mayor has the power to veto the motion. However, if all three of the council members vote to pass a motion then the mayor cannot veto it.

(e) List all sequential coalitions in the table below. Circle the pivotal player in each sequential coalition. (All boxes should be filled. If you're careful when filling in the table, part (f) will be easier)

(f) Compute the **Shapley-Shubik power distribution** for the system. Does the mayor really have any more power than the council members?

3. (12 pts) Tony, Carmela, Janice, and Bobby are playing Monopoly when Bobby quits over the introduction of the "Free Parking rule." The other three have already divided his money evenly amongst themselves, but now they must divide Bobby's three acquired properties (Reading Railroad, Virginia Avenue, Marven Gardens, and Park Place) fairly. They decide to use the **method of sealed bids**. Below is a table listing each player's bid.

	Tony	Carmela	Janice
Reading Railroad	$250	$300	$160
Virginia Avenue	$260	$130	$190
Marven Gardens	$280	$330	$300
Park Place	$500	$470	$460
Total			
Fair share			

(a) In the first allocation, who gets what <u>properties</u> (money is covered in (c))?

(b) Fill in the "total" and "fair share" rows of the table.

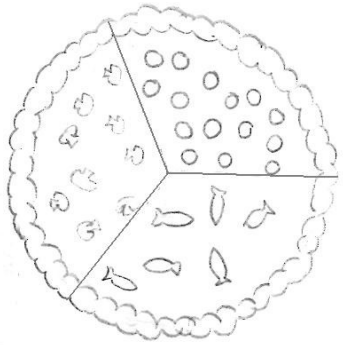
(c) In the first allocation, how much <u>money</u> does each player lose or receive?

(d) After the first allocation, how much of a surplus is there (i.e. how much is "in the pot")?

(e) What is each player's share of the surplus?

(f) What is the final allocation of properties and money? (Be sure to mention whether a player is losing or gaining money).

4. (18 Pts) Leonardo, Donatello, Michelangelo, and Raphael are dividing a $12 pizza. It's up to you whether these are the Teenage Mutant Ninja Turtles or the Renaissance artists. The pizza is divided evenly into Pepperoni, Anchovies, and Mushrooms sections (See picture).



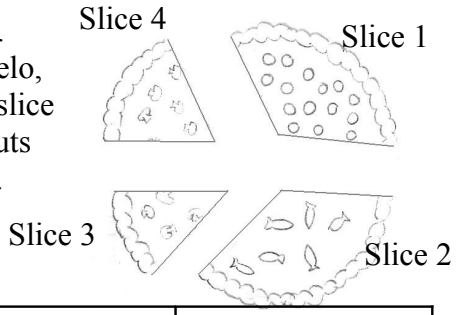
The hungry turtles/artists have the following preferences:

- Leonardo's values are as in the chart in part (a)
- Donatello is a vegetarian and will not eat pepperoni or anchovies
- Michelangelo is a party dude and likes all kinds of pizza equally
- Raphael hates anchovies, and likes pepperoni and mushrooms equally

(a) Complete the chart below, based on the preferences above.

	Pepperoni	Anchovies	Mushrooms
Leonardo	3	3	6
Donatello			
Michelangelo			
Raphael			

They decide to cut up the pizza using the **Lone Divider Method**. Leonardo is chosen as the divider, leaving Donatello, Michelangelo, and Raphael to be the choosers. Leonardo cuts the piece so that slice 1 is the pepperoni portion, slice 2 is the anchovies portion, and cuts the mushroom portion in half to form slices 3 and 4 (see picture).



(b) What is each slice worth (in $$) to each player?

	Slice 1	Slice 2	Slice 3	Slice 4
Leonardo				
Donatello				
Michelangelo				
Raphael				

(e) Did Leonardo **cut consistently with his value system**? Justify your answer.

(d) What is the **bid list** for each chooser?

(e) What would be a **fair distribution** for the four slices of pizza?

5. (25 pts) The Galactic Parliament has 11 seats, to be distributed among the three states (Andros, Baxteros, and Kristenos) in proportion to their population. Their populations (in billions) are summarized below

State	Andros	Baxteros	Kristenos
Population	54	243	703

(a) What is the **standard divisor** (round to two decimal places)? How can it be interpreted?

(b) Apportion the seats using **Hamilton's method**.

State	Population				Hamilton's Apportionment
A	54				
B	243				
K	703				
Total	1000				

(c) A new state, Wicketeria, joins the Galactic Alliance. Its population is 580, so 7 seats are added to Parliament. What is the new apportionment using **Hamilton's Method**? (Don't forget to recalculate the standard divisor)

State	Population				Hamilton's Apportionment
A	54				
B	243				
K	703				
W	580				
Total	1580				

(d) Did any **paradoxes** occur? If so, state which paradox and justify your answer.

(e) Citing the above paradox, Senator Xaphod Beeblebrox (from Kristenos) calls for a new apportionment scheme, which he calls the Beeblebrox Method. In a historical coincidence, it's

identical to Jefferson's Method. Use **Jefferson's Method** to apportion the 18 seats to the four states. (Timesaver: either the number 80 or 90 will come in handy)

State	Population			Jefferson's Apportionment
A	54			
B	243			
K	703			
W	580			
Total	1580			