Forward to: ENJOYMENT OF MATHEMATICS

The art form Mathematics is well-known to have a marketing problem. The great German mathematician Carl Friedrich Gauss famously said, "Mathematics is the queen of Science."[1] And yet this Queen, directly responsible for much of our modern existence and all its extranatural comforts, is greatly under appreciated, if not outright despised, by most of the educated public. I can't tell you how many people I've met who have told me, "I hate math". Upon further prodding, they'll admit that in fact they loved math up to Subject X (where X might be Calculus, or Trigonometry, or Algebra, etc), and then Subject $X + 1$ was the worst and they've hated math ever since. (Upon even further prodding, it turns out they really hated their *teacher* in Subject $X + 1$, hence never learned it properly, and everything thereafter was gobbledygook. Many of them also admit that as adults, they love struggling to solve difficult puzzles like Sudoku, KenKen, etc etc.)

You don't hear such statements nearly as often about a closely related subject, Physics. I've always wondered why. For one suggested explanation, Physics is not required in school, so the only people subjected to its rigors are there voluntarily. Another, perhaps, and much closer to the discussion at hand, is that a great number of Nobel laureate physicists write public accounts of their discoveries, wowing us with far away galaxies, quantum field theories, earth-like Goldilocks planets, black holes, quantum entanglement, and so on. The record for mathematicians of the highest caliber "wasting" their immense talents and invaluable time trying to reason with the general public is far, far sparser.

This is what makes the present book all the more remarkable. Here are two world-renowned mathematicians making a valiant attempt at showing the public some true gems of our beloved subject. Otto Toeplitz and Hans Rademacher were preeminent researchers who made fundamental contributions to 20th century mathematics. Toeplitz, born in 1881 in Breslau (modern-day Poland) to a Jewish family, cut his teeth at the University of Göttingen, which was the mathematical home of everyone from Gauss, to Riemann, to his own teachers, Hilbert, Klein, and Minkowski. When the Nazis came to power, Toeplitz was dismissed from his professorship at Bonn, and moved to the Hebrew University

[1] He continued, "and Number Theory" – which features heavily in the book before you – "is the queen of Mathematics."

in Jerusalem, where he died in 1940 from tuberculosis. Rademacher, about a decade Teoplitz's junior, hailed from the Hamburg area, and overlapped with his coauthor in Göttingen, where he earned his PhD in 1916. While Rademacher was racially acceptable to the Nazis, his pacifist views were not, and he was likewise dismissed from his position at the University of Breslau. Rademacher moved to the US to take an Assistant Professorship at the University of Pennsylvania (despite already having served a decade as a full professor in Germany), from which he retired in 1962, passing not long after in 1969. Their combined interest in and passion for popularizing mathematics, crystalized over many lectures to the public, came to fruition as *Von Zahlen und Figuren*, released in German in 1930, and translated into the text before you in 1957.

The topics presented within are not the recreational material one sometimes encounters; these are the "real deal." Already in the first chapter, we begin with Euclid's argument that the prime numbers are plentiful: for any finite collection of them, more exist; this is followed immediately with a contradistinction: one may have to wait an arbitrarily long while from one prime to the very next. Such issues are quite deep, being the subject of ongoing research in the present day, and yet their presentation here is completely "elementary," in the sense of not requiring any technical knowledge beyond the 5th grade. Instead what is needed is a penchant for finding such questions of interest, as well as the determination and stamina to chew over the arguments presented until they coalesce. This deep/elementary duality applies to most of the topics presented, ranging from geometry and arithmetic, to topology and graph theory. Some chapters, like that describing the Four-Color "Problem" (now a Theorem, resolved only in 1976), describe the state of things in the 1930s but of course not our present-day understanding. For some others, there are now new ways to showcase the topic to the public. For example, Chapter 25 discusses Curves of Constant Breadth; today someone wanting to learn about such can visit the National Museum of Mathematics in NYC and take a spin on some non-spherical coasters which nonetheless ride smoothly.

Overall, the topics seem to me likely on the advanced side for today's general audience. Maybe today we know more about what the general public can and cannot typically follow. Or maybe previous generations experienced so much more daily hardship that a few hours of struggle with a mathematical chapter (or just a paragraph, or even a sentence) would, by comparison, not seem insurmountable to them.

On the other hand, I'm reminded of a story Keith Devlin told me some time ago about the first time he wrote something and submitted it to a local paper. The editor called him back, and said he'd be delighted to have him write something, but what Keith had written was *way* too complicated for their audience; he'd need to craft something that people could read for five minutes over a cup of coffee (and when Keith's medium moved to radio, he was similarly told that his topic needed to be digestible while the Average Joe was pumping his gas). Now, Keith has been tremendously successful with communicating with the public. But the modern experience with much less "gatekeeping" and editorializing has shown several things about the public audience: (*i*) People are not just willing to watch some 30 minute sitcom or 2 hour movie, but they'll eagerly binge a 14 hour mega-saga on Netflix; (*ii*) People do not just listen to the radio while pumping gas, but will download 3-hour long Podcasts and follow incredibly convoluted storylines with dozens of complex characters and interactions; and perhaps most relevant to us here: (*iii*) People will gladly watch, by the millions and tens of millions, videos on very advanced graduate-level mathematics, when presented by masters of exposition like the YouTube channels 3blue1brown, Veritasium, and many others. All this is to say that, perhaps if this book were to be proposed today, its authors may be told by editors to water down the subjects and discussion; but since it has already achieved a significant amount of success, it has earned its right to be as it is. I sincerely hope that you, dear reader, will give its message the time required to struggle with the difficult, important, and timeless material; you will be greatly rewarded!

We seem to be in a bit of a Renaissance for math exposition, both in terms of mathematicians givings greater respect for expository writing, and in the general public appreciating books like these. Let us hope that another generation may be inspired by the writings within.

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