Syllabus for Vijay

Algebraic Geometry (major topic)

1) Affine, projective, quasi-projective varieties
Zariski topology, ideals of varieties, coordinate ring
Ring of regular functions (= \( A(X)[1/f] \) of \( U_f \))
Local ring at a point
Regular maps
Examples: Veronese varieties, Segre varieties, sub-varieties of these
Product of varieties (is actually categorical product)

2) Cones
Classification of quadrics
Projections (proof that these are varieties)
Morphisms are closed maps

3) Nullstellensatz (Weak and Strong)

4) Grassmannians
Plucker embedding
Subvarieties of Grassmannians (incident planes, joins, fano varieties,
Schubert Varieties)

5) Rational functions and rational maps
Graphs of rational maps
Every variety is birational to a hypersurface
Blowups, blowdowns

6) Dimension
Equivalence of various definitions
Basic computations (Grassmannians, products, cones, projections)
Intersection of a variety with a hypersurface
Fiber Dimension Theorem
Complete intersections
Dimensions of secant varieties, joins, flag manifolds, Schubert varieties

7) Hilbert Functions
\( h(m) \) is a polynomial for large \( m \)
Degree of polynomial = dimension of variety

8) Smoothness, Tangent Spaces
Variety is singular at point iff local ring is regular local ring
Singular locus of a variety is proper closed subset
Singular points of Schubert varieties

9) Sheaves, Sheafification, Etale Space
Schemes: Spec, Proj, and the Functor from Varieties to Schemes

References
Joe Harris, Algebraic Geometry: A First Course
Robin Hartshorne, Algebraic Geometry

Algebraic Topology (minor topic)
1) The Fundamental Group
   The Seifert-Van Kampen Theorem
   Classification of covering spaces
   Deck Transformations and group actions

2) Simplicial homology
   Singular homology
   Exact Sequence and Excision
   Cellular Homology
   Mayer-Vietoris Sequence

3) Cohomology ring
   Künneth formula
   Cup Product
   Statement of Poincare Duality

4) Vector bundles
   Grassmann manifolds
   Universal Bundles, Gauss Maps

References
Allen Hatcher, Algebraic Topology
Milnor, Stasheff, Characteristic Classes