Oral Exam Syllabus

1 Lie Groups

- Definition of a Lie group; examples (including classical Lie groups)
- Lie algebras and their relation to Lie groups
  - Exponential mapping
  - Adjoint and co-adjoint representation
- Representations of compact, connected Lie groups
  - Peter-Weyl theorem
  - Maximal Tori: existence, uniqueness up to conjugation, Weyl covering theorem, examples for classical groups.
  - Weyl group; action on maximal torus and its Lie algebra
  - Complexification; roots; positive roots; dominant alcove
  - Dynkin diagrams
  - Weight spaces, dominant weights
  - Highest weight theorem
- Formulae
  - Weyl integration formula
  - Weyl character formula
  - Dimension formula
- Homogeneous vector bundles
  - Induced representations
  - Frobenius reciprocity
• Borel-Weil theorem

2 Functional Analysis

• Banach spaces
  Examples ($L^p$ spaces, sequence spaces, direct sums, quotients)
  Linear functionals: duals, reflexive spaces, Hahn-Banach theorems
  Baire category theorem, Open Mapping theorem, Closed Graph theorem, Banach-Steinhaus (uniform boundedness) theorem
  Hilbert spaces (polarisation, adjoints, Riesz lemma)

• Topological devices
  Nets
  Compactness (Tychonoff theorem, Urysohn’s lemma, Stone-Weierstrass theorem)
  Banach-Alaoglu theorem

• Bounded operator theory
  Adjoints
  Spectrum
  Compact operators
  Fredholm alternative
  Spectral decomposition of compact, self-adjoint operators

• Differential operators and spectral theory
  Schwarz space
  Fourier transform
  Distributions
  Sobolev spaces
References

