1. Algebraic Number Theory
   (a) Invariants of number fields: rings of integers, discriminants, orders
   (b) Arithmetic of number fields: splitting of primes, ramification, Frobenius, class groups
   (c) Structure of units in number fields
   (d) Binary quadratic forms
   (e) Kronecker-Weber theorem
   (f) Ideles, adeles
   (g) Cebotarev density theorem, Artin reciprocity, class field theory
2. Analytic Number Theory
   (a) Analytic properties of L-functions
   (b) Primes in arithmetic progressions
   (c) Siegel zero problem
   (d) Prime number theorem and prime number theorem for arithmetic progressions
   (e) Large sieve inequality for multiplicative characters
3. Elliptic Curves
   (a) Elliptic curves over \( \mathbb{C} \)
   (b) Elliptic curves over finite fields, Hasse’s theorem
   (c) Hasse-Weil L-functions
   (d) Mordell-Weil theorem and descent on elliptic curves
   (e) Complex multiplication
4. Modular Forms
   (a) Modular forms for the full modular group and its congruence subgroups
   (b) Eisenstein series
   (c) Mellin transforms, L-series, converse theorems
   (d) Hecke operators
   (e) Modular forms of half-integer weight