Oral Qualifying Exam Syllabus

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1. Algebraic Topology
The Fundamental Group
The Seifert-Van Kampen Theorem
Covering Spaces
Lifting criterion/ Lifting properties
Deck Transformation group

Singular Homology
First homology group as the abelianization of the Fundamental Group
Relative homology
Exact Sequence and Excision
Betti numbers and Euler characteristics
Singular and Cellular Homology
Mayer-Vietoris Sequence

References
Allen Hatcher, Algebraic Topology
James Vick, Homology Theory: An Introduction to Algebraic Topology

2. Riemann Surfaces
Definition of Riemann Surfaces
Maps between Riemann Surfaces
Smooth surfaces

Cotangent spaces and 1-forms
2-forms and integrations
Analytic and meromorphic forms
De Rham cohomology for surfaces

Calculus on Riemann surfaces
Laplace operator and Harmonic functions
The Dirichlet norm

Weyl's lemma
Uniformization theorem
Classification of Riemann surfaces

References
Otto Forster, Lectures on Riemann Surfaces  
Simon Donaldson, Notes on Riemann Surfaces  

3. Riemannian Geometry  
Definition of Riemannian metrics  
Riemannian Length and Distance  
Space forms/Model spaces  
Three models of hyperbolic spaces  
The Levi-Civita Connection  
Curvature tensors and curvature identities  

Reference  
John M. Lee, Riemannian Manifolds: An Introduction to Curvature