Oral Qualifying Exam Syllabus

Michael Marcondes de Freitas

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Committee

- Daniel Ocone
- Roger Nussbaum
- Eduardo Sontag
- Eugene Speer

Major Topic: Ordinary Differential Equations and Monotone Systems

Basic Ordinary Differential Equations Theory

- Existence, uniqueness and continuous dependence on initial conditions and parameters
- Orbits and asymptotic limit sets
- Poincaré-Bendixson Theorem

Monotone Dynamical Systems

- The convergence criterion
- The limit set dichotomy
- Quasiconvergence results

Stability and Convergence

- Stability
- The order interval trichotomy
- Global results
- Generic convergence to equilibrium
- Unstable equilibrium and connecting orbits

**Competitive and Cooperative Differential Equations**
- The Kamke condition
- Positively invariant sets and monotone solutions
- Three-dimensional systems
- Alternative cones
- The Field-Noyes model

**Irreducible Cooperative System**
- Strong monotonicity
- Biochemical control circuits
- Stability and the Perron-Frobenius Theorem
- Competition and migration
- Smale's construction

**Minor Topic: Mathematical Biology**

**Deterministic Models**
- Growth models: exponential growth, logistical equation, chemostat
- Reducing number of parameters
- Steady states and linearized stability analysis
- Phase planes: vector fields, nullclines, global behavior
- Effect of drug on cells in an organ
- Compartment models
- Epidemiology models: immunization, STD models
- Chemical kinetics: chemical networks, quasi-steady state approximations, fast and slow behavior, singular perturbation analysis, inhibition, cooperativity
- Multi-stability
- Periodic Behavior
- Bifurcations
Stochastic Biochemical Models

- Stochastic models of chemical reactions
- The Chemical Master Equation
- Theoretical background and algorithms: Markov processes, interpretation of the master equation and propensity functions, the stochastic simulation algorithm, interpretation of mass-action kinetics
- Moment equations and fluctuation-dissipation formula
- Generating functions
- Conservation laws and stoichiometry

References

