Syllabus for Oral Exam

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Major topic: Partial Differential Equation

1. Four Important Linear PDEs
   1.1 Transport Equation
   1.2 Laplace’s Equation
      - Fundamental solution
      - Mean-value formula
      - Properties of harmonic functions (Maximum principles, smoothness)
      - Green’s function
   1.3 Heat Equation
      - Fundamental solution
      - Mean-value formula
      - Maximal principle (for bounded domain; for Cauchy problem)
      - Energy methods (for uniqueness for initial/boundary-value problem; for backward uniqueness)
   1.4 Wave Equation
      - d’Alembert’s formula
      - Solution by spherical means
      - Energy methods (for uniqueness for wave equation)
   1.5 Schrödinger Equation
      - Derivation of its fundamental solution

2. Sobolev Spaces
2.1 Holder spaces
2.2 Sobolev spaces
   Weak derivatives, definition of Sobolev spaces, elementary properties
2.3 approximation
2.4 extensions
2.5 Sobolev inequalities
   Gagliardo-Nirenberg-Sobolev inequality
   Morrey’s inequality
   General Sobolev inequality
2.6 Compactness
3. More General Second-Order Elliptic Equations
   3.1 Existence of weak solutions
      Definition of weak solution
      Lax-Milgram Thm
      Energy estimates
      Fredholm alternatives
   3.2 Regularity
      Interior regularity, boundary regularity
   3.3 Maximum principles
      Weak maximum principle
      Strong maximum principle

Minor topic: Numerical Solution for PDEs
   1. Approximation of function by polynomial and piecewise polynomial
Newton interpolation
Lagrange interpolation
Application to numerical differentiation and numerical quadrature
Error Estimates

2. Finite Difference Method for elliptic PDEs and heat PDEs
   discrete maximum principle
   discrete Green's function
   error estimates

3. Finite Element Methods for PDEs
   triangulation of domain
   barycentric coordinates
   variational formulations of elliptic PDE (standard Galerkin; mixed
   finite element methods; nonconforming methods)
   error estimates in $H^1$ and $L^2$
   solution of the discrete equations