1 Probability Theory

Probability Spaces
- Algebras and \(\sigma\)-algebras of collections of sets (Review)
- Probability spaces
- Probability spaces; elementary examples, coin tossing
- The monotone class theorem
- Independence and product spaces

Random Variables, Distribution Functions, and Expectation
- Random variables and distribution functions
- Laws of random variables and distribution functions
- Expected values
- Independence of random variables
- Some basic probability laws
- Convergence concepts for random variables

Large Number Laws for Sequences of Random Variables
- The weak law of large numbers
- The Borel-Cantelli lemma
- Strong large number laws
- Convergence of infinite series of independent random variables and applications

Convergence in Distribution and the Central Limit Theorem
- Motivation: the central limit problem
- Weak convergence and convergence in distribution
- Characteristic functions; uniqueness and smoothing
- The continuity theorem

Conditional Expectation
- Definition of conditional expectation
- Properties of conditional expectation

Martingale Theory in Discrete Time
- Definitions and examples
- Stochastic integrals in discrete time and a fundamental theorem of Martingale theory
- Optional stopping of Martingale
- Martingale inequalities
- Martingale convergence theorems
2 Combinatorics I & II

Basic Enumeration

- Basic Counting
- Recurrence Relations
- Generating Functions
- Principle of Inclusion–Exclusion

Partially Ordered Sets, Lattices

- Dilworth’s Thm
- Sperner’s Thm
- Geometric and Distributive Lattices
- Möbius Functions

Hypergraphs

- Block Designs
- Fisher’s Ineq. & Related
- Erdős–Ko–Rado Thm
- Kruskal–Katona
- Hamming Cube & Isoperimetry
- Turán’s Thm & Erdős–Stone
- Baranyai

Ramsey Theory

- Infinite Ramsey
- Finite Ramsey
- Van der Waerden’s Thm, Szemerédi’s Thm

3 Graph Theory

Matching Theory

- Hall/König & Applications
- Max Flow Min Cut
- Tutte's 1-factor Thm

Connectivity

- 2-connected graphs
- Menger’s Thm

Graph Coloring

- Vertex Colorings
- Edge Colorings

Planar Graphs

Matroids

4 Probabilistic Methods

Lin. of Expectation & “Alterations”

Second Moment Method

Lovász Local Lemma

Poisson Paradigm & Janson Ineq.

Correlation Inequalities