Oral Qual Syllabus

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1 Combinatorics

Basic Enumeration:
counting arguments ([vLW, Chapter 13])
binomial coefficients ([K07, 9/4-9/11])
recurrence relations and generating functions ([vLW, Chapter 14])
inclusion–exclusion ([K07, 9/21-9/25], [vLW, Chapter 10])
Stirling’s formula ([F, p. 52–54], [K07, 9/14])

Set Systems:
Sperner’s Theorem and the LYM Inequality ([Bol, 3], [K07, 10/5-10/9],
vLW, p. 54)
Kruskal-Katona ([Bol, 5], [K07, 11/13]), implies Erdős-Ko-Rado ([Bol, 7])
Erdős-Ko-Rado ([Bol, 7], [K07,11/9], [vLW, p. 56])
Fisher inequality ([Bol 10.8], [vLW, p. 222])
Frankl-Wilson, Ray-Chaudhuri-Wilson ([FW], [vLW, Thm 19.8, p. 222])

Correlation Inequalities:
Four Functions Theorem ([AS, 6.1], [K07, 12/7], [Bol, 19.1])
FKG Inequality ([AS, 6.2], [K07,12/4], [Bol, 19.5])
Harris-Kleitman ([AS, 6.3], [K07,11/30], [Bol, 19.9])
Shepp’s XYZ-theorem ([AS, 6.4], [K07,12/11], [S])

Ramsey Theory:
Ramsey’s Theorem for graphs and hypergraphs ([K07,10/19], [vLW, p. 28])
countable and uncountable Ramsey Theory ([Bol, 20], [GRS, 1.1–1.2])
probabilistic lower bounds, upper bounds ([AS, p. 16, 25, 67], [vLW, p. 30])
Van der Waerden’s Theorem ([GRS, 2.1])
Hales-Jewett ([GRS,2.2])

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2 Graph Theory

Matching Theory:
Hall's theorem ([Bol, 2.2], [D, 2.1.2], [K07, 10/12], [vLW, 5])
matching algorithm: Augmenting Paths ([D, 2.1.2, second proof])
König's theorem ([D, Thm 2.1.1], [K07, 10/12])
König is equivalent to Dilworth's Theorem ([K07, 10/5, 10/12])
Gale-Shapley Algorithm for stable matchings ([D, Thm 2.1.4])
Tutte's Theorem on 1-factors ([D, Thm 2.2.1])

Spanning Trees:
Greedy algorithm for maximal spanning trees (matroid result)
Cayley's Theorem, Prüfer Codes ([vLW, 2.1, proof 1])

Planarity:
Euler's Formula ([D, Thm 4.2.9], [vLW, 33.3])
Kuratowski ([D, 4.4], [vLW, 33.2])
Wagner's Theorem ([D, 4.4.6])

Coloring:
Chromatic and Edge Chromatic Numbers
5 color theorem ([D, Prop 5.1.2], [vLW, 33.6])
Brook's Theorem ([D, Thm 5.2.4], [vLW, 3.1])
Vizing's Theorem ([D, Thm 5.3.2])

Extremal Problems:
Turan's Theorem ([vLW, 4], [D, p. 165], [AS, p. 91-92])
Statement of Regularity Lemma ([D, p. 176])
Erdős-Stone Theorem ([D, 7.5])
Triangle Removal Lemma (given Regularity Lemma)
Proof of Roth's Theorem (given Regularity Lemma)

Matroids:
Definitions, multiple characterizations ([Wal, 1])
Greedy Algorithm ([Wal, 19.1])
Covering and Packing Theorems ([Wal, 8.1-8.4])
Matroid Intersection Theorem, implies König's Theorem ([Wal, 8.5])
Transversals and generalizations of Hall's Theorem ([Wal, 7.1-7.4])
3  Probabilistic Methods

Basics:
Linearity of Expectation ([AS, Chapter 2], [K07, 10/23])
Bonferroni Inequalities ([Dur, I.1.3], [K07, 9/25])
common distributions ([Dur, I.1.5-1.7], [K07, 9/25])
conditional probability, law of total probability ([K07, 11/9])
Chernoff bounds ([AS, Apdx. A], [K07, 11/2])
Chebyshev Inequality ([AS, p. 41], [Dur, I.3.7], [K07, 10/26])

Allocation Method:
general procedure ([AS, 3.1])
basic examples:
  independent sets, packing, triangles in the unit square ([AS, 3.2-4])
application to Property B (hypergraph coloring) ([AS, 3.5])

Second Moment Method
general procedure ([AS, 4.1], [K07, 10/26])
application to threshold functions ([AS 4.4], [K07, 10/26-10/30])
Moment Method for eigenvalues of random matrices ([V08, 2/18])

Lovasz Local Lemma:
Symmetric and general versions ([AS, 5.1], [K07, 10/30-11/2])
Ramsey lower bounds ([AS, 5.3])
Hypergraph Coloring ([AS, 5.2], [K07, 10/30])

Martingales:
Definitions, Azuma’s Inequality ([AS 7.1-2], [V08, 4/3, 4/10])
application to chromatic number ([AS, 7.3])
Talagrand’s inequality ([AS, 7.5-7.6], [V08, 3/10, 3/27, 4/3])
comparing Talagrand’s and Azuma’s inequalities ([AS, 7.7], [V08, 3/10])
longest increasing subsequence problem ([AS, 7.7])

4  Probability Theory

Basics, Large Number Laws:
probability spaces ([Dur, I.1])
random variables([Dur, I.2])
expectation ([Dur, I.3])
independence of random variables ([Dur, I.4])
weak law of large numbers ([Dur, I.5])
Borel-Cantelli lemma ([Dur, I.6])
convergence concepts for random variables ([Dur, I.7])
Kolmogorov's 0-1 law ([Dur, I.7.1])
Kolmogorov's three series theorem ([Dur, I.7.4])
strong law of large numbers ([Dur, I.8])

**Central Limit Theorem:**
De Moivre-Laplace theorem ([Dur, II.1])
weak convergence and convergence in distribution ([Dur, II.2])
characteristic functions ([Dur, II.3])
continuity theorem, characteristic functions ([Dur, II.3.6])
Lindeberg-Feller central limit theorem ([Dur, II.4])
Berry-Esseen Theorem ([Dur, II.4])

**Martingales:**
conditional expectation ([Dur, IV.1])
definition of (sub)(super) martingales ([Dur, IV.2])
stopped times ([Dur, III.1])
application to random walks ([Dur, III])
Doob’s upcrossing inequality ([Dur, IV.2.9])
martingale convergence theorems ([Dur, IV.2.10])

**References**

