Oral Qualifying Exam Syllabus
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Committee: Profs. R. Wilson, R. Goodman, V. Retakh, E. Taft

Major Topic: Noncommutative Algebra

1. General noncommutative ring theory ([Lam] §§1-4, 7-9)
   - Semi-simple modules and rings
   - Wedderburn-Artin Theory
   - Jacobson Radical
   - modules over kG/representations of finite groups, characters
   - linear groups: Burnside’s Theorem

2. Quasideterminants ([GGRW])
   - definition - in terms of inverses and recursive
   - properties - e.g. row/column relations, Sylvester’s theorem
   - applications - e.g. Vandermonde quasideterminant, Vieta theorem, symmetric functions, quasi-Plucker coordinates

3. Algebras related to roots of equations
   - $Q_n$, $A(\Gamma)$ - definition, describe linear basis[GRSW]
   - Bergmans Diamond Lemma ([Bergman] §§1-3)
   - factorization of twisted polynomial rings: remainder and product theorems, definition and example of Wedderburn polynomials ([LL] §§1-2; [LL2] §§1-3)
   - Koszul algebras: definition, dual of, Hilbert series of ([Froberg] §§1-2)

4. Lie Algebras ([GW] Ch. 1-2) ([Humphreys] §§1-8)
   - Lie group and Lie algebra correspondence
   - classification of finite dimensional semisimple algebras over the complex numbers
   - classification of irreducible representations by highest weight (isomorphism)
   - some explicit examples
   - PBW Theorem

Minor Topic: Hopf Algebras

- Definition of Hopf algebra (coalgebra, bialgebra, antipode)
- Coideals and comodules
- Duality - $A^0$, $C^*$
- Definition of integrals and smash product
- H-module algebra and coalgebra
- Fundamental Theorem of Coalgebras
References


GRSW, Gelfand, Israel, Vladimir Retakh, Shirlei Serconek, and Robert Wilson. "On a Class of Algebras Associated to Directed Graphs."


