Oral Qualifying Exam Syllabus
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1. Combinatorics and Graph Theory
   1.1 Combinatorics:

   **Counting and set theory:** binomial coefficients, recurrence relations, generating functions, inclusion-exclusion, Stirling's formula, Stirling number, Erdős-Ko-Rado, Fisher's inequality, Ray-Chaudhuri-Wilson.

   **Lattice and poset:** Distributive and geometry lattices, Birkhoff representation theorem, matroid, Mobius function, Weisner's theorem, Dilworth, Sperner, LYM inequality, linear extension of poset, dimension of poset.

   **Correlation inequality:** Harris-Kleitman, Fortuin-Kastekeyn-Ginibre, Ahlswede-Daylein, Sheppe XYZ.

   **Matching theory:** Hall's thm, König's thm, fractional cover and fractional matching, matching polytope.

   **Ramsey Theory:** Ramsey, infinite Ramsey, König's Lemma, Van de Waerden

   1.2 Probabilistic Methods:

   **Basis:** linearity of expectation, Markov's inequality, Chebyshev's inequality, Chernoff bound, binomial and Poisson distribution.

   **Alternations:** Ramsey, Independent number, graph with high girth and high chromatic number.

   **Second moment method:** Threshold function, subgraph, clique number.

   **Lovasz local lemma:** Symmetric and general versions, application to Ramsey.

   **Poisson Paradigm:** Janson's Inequality and application on chromatic number of $G_{n,1/2}$. Brun's sieve and application on EPIT.

   **Martingales:** Edge and vertex exposure, Azuma's inequality and application on chromatic number.

   1.3 Graph Theory:

   **Matching:** Tutte's thm, stable matching

   **Connectivity:** Menger's Thm, Max Flow/Min Cut, structure of 2-connected
graphs, minimal spanning tree, Kruskal's algorithm.

**Extremal Problems:** Turan's Theorem, Regularity lemma and its application on the Erdös-Stone Theorem, Chvatal-Rodl-Szemerédi-Trotter

**Planarity:** Euler's Formula, Kuratowski, Wagner

**Coloring:** Chromatic and Edge Chromatic Numbers, Brook's Theorem, Vizing's Theorem, Thomassen's Theorem, 5-color theorem, Galvin's Theorem, perfect graphs: definition and statements of theorems

2. Computational Complexity

**P v. NP:** Definitions, reducibility, the Cook-Levin Theorem, NP completeness of SAT, Independent set, 0/1 integer programming, coNP, what if P=NP

**Diagonalization:** Ladner's Theorem, Oracle Turing Machines, Baker-Gill-Solovay Theorem

**Space-bounded complexity:** definitions, configuration graph, PSPACE completeness of TQBF, NL completeness of PATH, Savitch's theorem, Immerman-Szelepcsényi Theorem

**Polynomial hierarchy:** Definitions of \( \Sigma_i \), \( \Pi_i \), complete problems, ATM, AP=PSPACE, Time/Space tradeoff for SAT

**Circuits:** \( P \subseteq P/poly \), CKT-SAT and alternate proof of Cook-Levin, Characterization of \( P/poly \) as TMs with advice, Karp-Lipton Theorem, Meyer's Theorem, existence of hard functions, Nonuniform Hierarchy Theorem, definitions of \( NC_i \), \( AC_i \)

**Randomization:** Definitions of RP, BPP and ZPP, \( \exists \exists P \cap \forall \forall coRP \), Error reduction, Sipser-Gacs Theorem, \( BPP \subseteq P/poly \), \( BPP \subseteq \Sigma_2^p \cup \Pi_2^p \), randomized reductions and definition of \( BP \cdot NP \)

**Interactive Proofs:** definitions, dIP=NP, GNI \( \in AM \), NP completeness of \( GI \) implies \( \Sigma_2 = \Pi_2 \), IP=PSPACE

**Decision Trees:** Decision tree complexity, 0- and 1-certificates, certificate complexity, randomized decision tree complexity, sensitivity, block sensitivity, degree, relationships between \( s(f), bs(f), C(f), D(f), \text{deg}(f), R(f) \)

**Communication Complexity:** Fooling sets, tiling lower bound, rank lower bound, Discrepancy, \( \varepsilon(f) \)