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1. Modular Forms

(a) The Modular Group
   i. $SL(2, \mathbb{Z})$ and Congruence Subgroups
   ii. Fundamental Domains for $SL(2, \mathbb{Z})$ and congruence subgroups
   iii. Cusps and Elliptic Points
   iv. The invariant measure of $\mathcal{H}$ under $SL(2, \mathbb{R})$.

(b) Modular forms
   i. Modular and cusp forms
   ii. Fourier expansions
   iii. The dimensions of $M_k(\Gamma(1))$ and $S_k(\Gamma(1))$
   iv. Eisenstein Series, the Dedekind $\eta$ function, $\Delta$, and the Jacobi triple product formula
   v. The Petersson inner product on $S_k(\Gamma(1))$.
   vi. The L-functions for modular form and functional equations

(c) Hecke Operators
   i. The slash and Hecke operators on holomorphic functions
   ii. Coset representatives for $SL(2, \mathbb{Z}) \backslash M_n(\mathbb{Z})$
   iii. Commutativity and self-adjointness of the Hecke operators
   iv. Hecke eigenforms and Fourier coefficients
   v. Euler products for Hecke eigenforms

(d) The Rankin-Selberg Method
   i. The nonholomorphic Eisenstein series
   ii. Analytic continuations and Euler products for the product of two modular forms

2. Ergodic Theory

(a) Transformations on probability spaces
   i. Measure preserving transformations
   ii. Invertible Extensions
   iii. The unitary operator and its spectral properties
   iv. Poincaré Recurrence
   v. Strong mixing
   vi. Weak mixing and equivalent definitions
   vii. Ergodic transformations and equivalent definitions
   viii. Invariant measures for continuous maps
   ix. Unique Ergodicity
   x. Weyl’s equidistribution criterion
   xi. An ergodic proof of Weyl’s theorem on equidistribution of polynomial sequences

(b) Ergodic Theorems
   i. Mean ergodic theorem
ii. Maximal inequality and maximal ergodic theorem
iii. Birkhoff’s pointwise ergodic theorem

(c) Continued Fractions
   i. The Gauss map and Gauss measure
   ii. Consequences of ergodicity of the Gauss map
   iii. Badly approximable numbers

(d) Geodesic Flow
   i. Hyperbolic geometry
   ii. Geodesic and Horocycle Flow
   iii. Ergodicity of geodesic flow

3. Analytic Number Theory

   (a) Poisson summation
   (b) The Mellin transform and the \( \Gamma \) function
   (c) The Phragmen-Lindelof principle and convexity bounds
   (d) The Riemann \( \zeta \) function and Dirichlet L-functions
      i. Euler products
      ii. \( \theta \) functions
      iii. Analytic continuation and functional equations
   (e) Dirichlet’s theorem on primes in arithmetic progressions.
   (f) The Prime Number Theorem.