Syllabus
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I. Major Topic: Partial Differential Equations

1. First order partial differential equations: Wave-particle duality
   - Space of 1-jets
   - Standard contact structure and its associated symplectic form in the space of 1-jets
   - Characteristic direction
   - Reduction of a nonlinear first order PDE to a system of nonlinear first order ODE’s
   - Local existence

2. Second order elliptic equations
   (a) Constant coefficient case: Laplace equation
       - Mean value formula
       - Maximum principle
       - Uniqueness
       - Regularity
       - Harnack inequality
       - Representation formulas
   (b) Sobolev spaces
       - Completeness, reflexivity, density, extensions
       - Gagliardo-Nirenberg-Sobolev inequalities
       - Morrey inequality
       - Poincaré inequality
       - Rellich-Kondrachov compact embedding theorem
       - Border line case: $W_0^{1,n} \hookrightarrow ?$
   (c) Weak solutions: $L^2$ theory
       - Lax-Milgram theorem and Fredholm alternative
       - Existence and uniqueness
       - Regularity

1
• Maximum principle

3. **Second order elliptic systems: Minimizers of quadratic functionals**
   
   (a) Convexity
   • Elliptic condition and convexity
   • Legendre-Hadamard condition and quasi-convexity
   
   (b) Existence
   • Coercivity
   • Weak lower-semicontinuity
   
   (c) Regularity in 2-d: Morrey theorem
   (d) Regularity for linear elliptic systems
   (e) Regularity for continuous minimizers

II. **Minor Topic: Riemannian Geometry**

1. Differentiable manifolds
   • Tangent and cotangent bundles
   • Vector fields and forms
   • Bracket
   • Immersions and embeddings

2. Riemannian metrics
   • Length
   • Volume

3. Connections
   • Affine connections
   • Riemannian connections
   • Covariant derivative along a curve

4. Geodesics
   • Geodesic equation
   • Geodesic flow
   • Exponential map
   • Gauss lemma
   • Minimizing properties
5. Curvatures
   - Curvature
   - Bianchi identities
   - Sectional curvature
   - Ricci curvature
   - Scalar curvature

6. Jacobi fields
   - Jacobi equation
   - Conjugate points

7. Completeness
   - Hopf-Rinow theorem
   - Hadamard theorem

References


