Syllabus for Oral Qualifying Exam

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I. Mean Curvature Flow
   1. Definition of the Mean Curvature Flow:
      • First Variation of the Area Funtional
      • Special Solutions
      • Short Time Existence of the Flow

   2. Evolution of Geometric Quantities:
      • Maximum Principle
      • Comparison Principle
      • Evolution of Curvature
      • Consequences of Evolution Equations
      • Convexity Invariance

   3. Monotonicity Formula and Type I Singularities:
      • The Monotonicity Formula and Integral Estimates
      • Type I Singularities and the Rescaling Procedure
      • Analysis of Singularities
      • Hypersurfaces with Nonnegative Mean Curvature

   4. Type II Singularities:
      • Hamilton’s Blow-up
      • Hypersurfaces with Nonnegative Mean Curvature
      • Hamilton’s Harnack Estimates for Mean Curvature Flow

   4. Regularity Theory at the First Sinfular Time:
      • Lower Bound on Area Ratio / Clearing Out Lemma
• White’s Gap Theorem
• Brakke’s Regularity Theorem under Area Continuity and Unit Density Hypothesis

II. Partial Differential Equations

1. Sobolev Spaces
   • Holder and Sobolev spaces
   • Approximation
   • Extensions
   • Traces
   • Sobolev inequalities
   • Compactness

2. Laplace’s Equation
   • Fundamental Solution
   • Mean-Value Formulas
   • Properties of Harmonic Functions
   • Green’s Function
   • Energy Methods

3. Second-Order Elliptic Equations
   • Existence of Weak Solutions
   • Regularity
   • Maximum Principles
   • Eigenvalues and Eigenfunctions

4. Heat Equation
   • Fundamental Solution
   • Mean-Value Formula
   • Properties of Solutions
   • Energy Methods

5. Second-Order Parabolic Equations
   • Existence of Weak Solutions
   • Regularity
   • Maximum Principles
References

[Ec] Klaus Ecker, Regularity Theory for Mean Curvature Flow
[Ev] Lawrence C. Evans, Partial Differential Equations