Problem 1 (15 points): Let \( f(x) = x^2 + x - 5 \). Prove by definition that \( f(x) \) is uniformly continuous over any closed interval \([-R, R]\) with \( R > 0 \). Also prove that \( f(x) \) is not uniformly continuous over \(( -\infty, \infty )\).
**Problem 2** (15 points): Let $f(x)$ be a function over $(0, 1]$. Assume that $f(x)$ is uniformly continuous over $(0, 1]$. Prove that $\lim_{x \to 0^+} f(x)$ exists.
Problem 3 (10 points): Suppose that \( f(x) \) is continuous over \((-1, 1)\). Assume that \( \lim_{x \to 0} f(x) = 1 \). Prove that there is a \( \delta > 0 \) such that \( f(x) > 1/2 \) for \( x \in (-\delta, \delta) \).
Problem 4 (15 points): Let $f(x)$ be a differentiable function over $[0, 1]$. Suppose that $f(0) = f(1) = 0$. Prove that $f'(x) - 2f(x)$ must have a zero inside $(0, 1)$. Namely, there is a $c \in (0, 1)$ such that $f'(c) - 2f(c) = 0$. 
Problem 5 (15 points): (a). Let \( f(x) \) be a decreasing function over \((a, b)\). Show that \( f'(x) \leq 0 \).

(b). Let \( f(x) \) be defined by \(-x + 2x^2 \cos\left(\frac{1}{x}\right)\) for \( x \neq 0 \) and define \( f(0) = 0 \). Show that \( f'(0) < 0 \).

However, for any \( \delta > 0 \), \( f(x) \) is not a decreasing function over \((-\delta, \delta)\).
**Problem 6** (15 points): (A). For any $a < b$, prove that the closed interval $[a, b]$ is compact. (B). Show that the open interval $(a, b)$ is not compact.
Problem 7 (10 points): Suppose that $f(x)$ is differentiable over $(a, b)$ except possibly at $c \in (a, b)$. Suppose that $f(x)$ is continuous over $(a, b)$ and $\lim_{x \to c} f'(x)$ exists. Prove that $f'(c)$ also exists.
Problem 8 (10 points): (A). Suppose $f(x)$ is differentiable in $(a - \delta, b + \delta)$ for a certain $\delta > 0$. Assume that $f'(a) > 0$ and $f'(b) < 0$. Show that there is a point $c \in (a, b)$ such that $f'(c) = 0$. (B). Construct a function $f$ over $(-2, 2)$ which cannot be the derivative of any function defined over $(-2, 2)$. 