Math 152  Final Exam Information

Things to Know

The final exam is a 3 hour comprehensive exam. The following summary lists the general subject areas that will appear on the exam. It does not detail every single type of problem that may appear. Students are expected to know everything that was covered during the semester, whether it is included here or not.

Simplifying: All answers should be simplified. For example, square roots and trig functions should be evaluated whenever possible. You may leave an inverse hyperbolic function in your answer, but not composed with another hyperbolic function.

A. Integration Techniques: all of them, including hyperbolic integrals and hyperbolic substitution.

Often trig identities are needed when integrating. It is strongly recommended that you memorize the following identities, which are only a fraction of what is required in this course.

\[
\begin{align*}
\sin^2 \theta + \cos^2 \theta &= 1 \\
\tan^2 \theta &= \sec^2 \theta - 1 \\
cos(2\theta) &= 2 \sin \theta \cos \theta \\
\sin^2 \theta &= \frac{1}{2} \left(1 - \cos(2\theta)\right) \\
\sin^2 \theta &= \frac{1}{2} \left(1 + \cos(2\theta)\right) \\
\cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta
\end{align*}
\]

In addition to the material covered in this course, students are expected to know all differentiation techniques, identities, theorems, and definitions from Calculus I. In particular, students must know the definitions and the derivatives of hyperbolic functions, as well as some of their basic identities.

Hyperbolic Functions – Definitions

\[
\begin{align*}
\sinh x &= \frac{e^x - e^{-x}}{2} \\
\cosh x &= \frac{e^x + e^{-x}}{2} \\
\tanh x &= \frac{\sinh x}{\cosh x} \\
\coth x &= \frac{\cosh x}{\sinh x} \\
\sech x &= \frac{1}{\cosh x} \\
\csch x &= \frac{1}{\sinh x}
\end{align*}
\]

Derivatives of Hyperbolic Functions

\[
\begin{align*}
(\sinh x)' &= \cosh x & (\cosh x)' &= \sinh x & (\tanh x)' &= \sec^2 x \\
(\csch x)' &= -\coth x \csch x & (\sech x)' &= -\tanh x \sech x & (\coth x)' &= -\csch^2 x
\end{align*}
\]

Hyperbolic Identities

\[
\begin{align*}
\cosh^2 x - \sinh^2 x &= 1 \\
\cosh(2x) &= \cosh^2 x + \sinh^2 x \\
\sinh(2x) &= 2 \sinh x \cosh x \\
1 - \tanh^2 x &= \sech^2 x \\
\cosh^2 x &= \frac{1}{2} \left(1 + \cosh(2x)\right) \\
\sinh^2 x &= \frac{1}{2} \left(\cosh(2x) - 1\right)
\end{align*}
\]
B. **Improper Integrals**: Determine whether an improper integral of type I or type II converges; evaluate improper integrals.

C. **Numerical Approximations**: Midpoint, Trapezoid, and Simpson’s Rules. (Error bounds for these approximations will be given.)

D. **Areas and Volumes**: Area between curves, Average Value of a function, Mean Value Theorem for Integrals, Volume as an Integral of a Cross Sectional Area, Volumes of Revolution (Shells, Washers), Arc Length, Surface Area.

E. **Sequences and Series**:
   - **Sequences**: Determine whether a sequence converges, Squeeze Theorem, L’Hôpital’s Rule, Geometric Sequences, hierarchy of sequences, Theorem 6 (pg 519).

<table>
<thead>
<tr>
<th>Sequence Hierarchy</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ln n ) (&lt;\cdots&lt;\sqrt[3]{n} ) (&lt;\sqrt{n} ) (&lt;n ) (&lt;n^2 ) (&lt;n^3 ) (&lt;\cdots&lt;2^n ) (&lt;3^n ) (&lt;\cdots&lt;) (n! ) (&lt;n^n )</td>
</tr>
<tr>
<td>( \frac{1}{\ln n} ) (\cdots&gt;\frac{1}{\sqrt[3]{n}} ) (&gt;\frac{1}{\sqrt{n}} ) (&gt;\frac{1}{n} ) (&gt;\frac{1}{n^2} ) (&gt;\frac{1}{n^3} ) (&gt;\cdots&gt;\frac{1}{2^n} ) (\frac{1}{3^n} ) (&gt;\cdots&gt;\frac{1}{n!} ) (&gt;\frac{1}{n^n} )</td>
</tr>
</tbody>
</table>

- **Series**: Geometric and \(p\)-series, Partial Sums, convergence tests (Divergence Test, Limit Comparison, Direct Comparison, Integral, Alternating Series test, Ratio, and Root tests), absolutely and conditionally convergent series, Error bound for Alternating Series.

- **Power Series**: Determine whether a power series converges, radius and interval of convergence, differentiation and integration of power series.

- **Maclaurin/Taylor Polynomials and Series**: Find the Taylor/Maclaurin Series of a function directly from the definition, OR from other known Taylor/Maclaurin Series expansions: \(\sin x, \cos x, e^x, \ln(1+x), \frac{1}{1-x}, \arctan(x)\). Term-by-term integration or differentiation of a known series expansion, determine the radius and interval of convergence, Binomial series.

F. **Parametrizations & Polar Coordinates**: Parametrizations of lines and circles, equation of a tangent line, area under a parametric curve, arc length of a parametric curve, surface area of revolution of a parametric curve. Polar coordinates of a point, polar equations of circles and lines, converting a polar equation to rectangular one, area in polar coordinates, area between curves in polar coordinates, arc length in polar coordinates.

G. **Complex Numbers**: Rectangular and Polar coordinates, polar form, exponential form, rectangular form, arithmetic of complex numbers, Euler’s Formula, De Moivre’s Formula, complex conjugates, modulus, polar or exponential to rectangular form and vice versa, plotting complex numbers, determine the \(n\)th roots of a complex number, geometric picture of the \(n\)th roots of a complex number.