Math 152, Midterm 1 – Review Problems

Do not assume that the first midterm problems will be similar to any of the problems below. Your midterm may contain questions that do not resemble any of the questions on this review.

Textbook Problems

Completing all WebAssign assignments is not sufficient preparation for the midterm. In particular, you will need to show all necessary steps on the exam, not just give an answer. It is strongly recommended that you work out all textbook problems listed on the department's Math 152 website.

Non-Textbook Problems

1. Find the area of the region in the first quadrant bounded by the curves $y = \sec x$, $y = \tan x$, $x = 0$, and $x = \pi/6$.

2. Find the area of the region bounded by the graphs of $y = \ln x$, $y = 1 - x$, and $y = 2$.

3. Let $\mathcal{R}$ be the region under the graph of $f(x) = \dfrac{1}{1+x^2}$ over the interval $(-\infty, \infty)$. **(a)** Find the area of $\mathcal{R}$. **(b)** Find the volume of the solid obtained when $\mathcal{R}$ is rotated around the x-axis.

4. Find all values of c in the interval $[-1, 1]$ which have the following property: $f(c)$ is the average value of f on $[-1, 1]$, where $f(x) = \sqrt{1 - x^2}$. Which theorem guarantees the existence of at least one such c?

5. Consider the solid whose base is the unit circle $x^2 + y^2 = 1$ and whose vertical cross sections perpendicular to the x-axis are rectangles of height $f(x) = |x|$. Find the volume of this solid.

6. Let $\mathcal{R}$ denote the region bounded by the x- and y-axes, and the graph of $f(x) = \sqrt{4 - x}$. **(a)** Find the volume of the solid generated by rotating $\mathcal{R}$ around the x-axis. **(b)** Find the volume of the solid generated by rotating $\mathcal{R}$ around the y-axis.

7. Consider a right circular cone with height H and base of radius R. Find the volume of this cone in two different ways: **(a)** using disks, **(b)** using shells.

8. Consider the triangular region in the xy-plane which is bounded by the lines $x = 3$, $y = 2$ and $y = 2(x - 3)$. A solid is obtained when this region is rotated about the line $x = 1$. Find the volume of this solid in two different ways: **(a)** using washers, **(b)** using shells.

9. Evaluate the following indefinite integrals using substitution.

a) $\displaystyle\int \cot x \, dx$ b) $\displaystyle\int \tanh x \, dx$ c) $\displaystyle\int \coth x \, dx$ d) $\displaystyle\int \tan x \, dx$

10. Evaluate the integral $\displaystyle\int \tan x \, dx$, using the substitution $u = \sec x$.

11. **(A)** Evaluate $\int \csc x \, dx$ using the substitution $u = \csc x + \cot x$. **(B)** Evaluate $\int \csc x \, dx$ using the substitution $u = \csc x - \cot x$. **(C)** Show that these two answers are equivalent.

12. Evaluate $\int \tan x \sec^4 x \, dx$ in two different ways.

13. Evaluate the following integrals using integration by parts.

a) $\int \sin^2 x\,dx$ b) $\int \cos^2 x\,dx$ c) $\int \sec^3 x\,dx$

14. Evaluate the following integrals.

a) $\displaystyle\int \frac{1}{x^2-8x+16}\,dx$

b) $\displaystyle\int \cos^2 x \tan^3 x\,dx$

c) $\displaystyle\int (\ln x)^3\,dx$

d) $\displaystyle\int \sqrt{9+x^2}\,dx$

e) $\displaystyle\int \sqrt{9-x^2}\,dx$

f) $\displaystyle\int \frac{\ln(\sqrt{x})}{\sqrt{x}}\,dx$

g) $\displaystyle\int \frac{3x^2-3x-2}{(x^2-1)(x-1)}\,dx$

h) $\displaystyle\int \frac{x^2+3x}{(x^2+1)(x+1)}\,dx$

i) $\displaystyle\int \frac{x^2}{(1-9x^2)^{3/2}}\,dx$

j) $\displaystyle\int e^{-x}\cos x\,dx$

k) $\displaystyle\int \frac{\sqrt{1+x^2}}{x}\,dx$

l) $\displaystyle\int \frac{dx}{1+e^x}$

15. Evaluate the integrals. Use the Change of Variable formula when substitution is used in definite integrals.

a) $\displaystyle\int_3^5 \frac{1}{x^2-8x+17}\,dx$

b) $\displaystyle\int_7^{11} \frac{1}{x^2-8x+15}\,dx$

c) $\displaystyle\int_{-\sqrt{3}}^{1/\sqrt{3}} x\arctan x\,dx$

d) $\displaystyle\int_{-1/3}^{1/3} \frac{x^2}{\sqrt{4-9x^2}}\,dx$

e) $\displaystyle\int_0^{\pi/2} \sqrt{1-\cos(4x)}\,dx$

f) $\displaystyle\int_0^{\sqrt{\pi}} x^3\sin(x^2)\,dx$

g) $\displaystyle\int_{-\pi/6}^{\pi/6} x\tan^2 x\,dx$

h) $\displaystyle\int_1^e \frac{\ln x}{x^2}\,dx$

i) $\displaystyle\int_1^{e^2} \frac{5-\ln x}{x(2+\ln x)}\,dx$

j) $\displaystyle\int_2^3 \frac{1}{\sqrt{4x-x^2}}\,dx$

k) $\displaystyle\int_{1/9}^{1/4} \frac{1}{\sqrt{x}(x-1)}\,dx$

l) $\displaystyle\int_{-1}^{1} \frac{e^{\arctan x}}{1+x^2}\,dx$

16. Let a and b be nonzero constants, where $a^2 \neq b^2$. Evaluate $\displaystyle\int \cos(ax)\sin(bx)\,dx$ using integration by parts twice.

17. Evaluate the following integrals or show that they do not converge. Use the Change of Variable formula when substitution is used in definite integrals.

a) $\displaystyle\int_1^\infty \frac{\ln x}{x^7}\,dx$ b) $\displaystyle\int_e^\infty \frac{1}{x\ln^2 x}\,dx$ c) $\displaystyle\int_4^\infty \frac{dx}{(2x+1)(3x+1)}$

18. Determine whether the integrals $\displaystyle\int_1^\infty xe^{-x^4}\,dx$, $\displaystyle\int_3^\infty \frac{dx}{\ln x-1}$, and $\displaystyle\int_1^\infty \frac{\cos^2 x}{x^3}\,dx$ converge.

19. Show that $\displaystyle\int_0^1 \frac{e^x}{x}\,dx$ diverges.

20. Evaluate the integral

$$\int \frac{x^2}{(x^2+1)^{3/2}}\,dx$$

using **(a)** trigonometric substitution, and **(b)** the hyperbolic substitution $x = \sinh t$.