Math 16:640:519
Selected Topics in Differential Equations – Spring 2015

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Summary: The goal of the course is to introduce Ph.D. students to non-linear evolutionary partial differential equations and their applications to problems in applied mathematics, differential geometry, and mathematical physics. Our course is motivated by the fact that a good understanding of non-linear parabolic partial differential equations is essential for many applications. Well-known examples include the Ginzburg-Landau equation, harmonic map heat flow, Navier-Stokes equation, Ricci curvature flow, Yamabe curvature flow, and Yang-Mills gradient flow. The Ricci curvature flow equation was used by Perelman to solve the Poincaré Conjecture in dimensional three. Many of these non-linear evolutionary equations can be very profitably analyzed as gradient-flow equations for an energy functional and this is the approach we shall take in this course. The course will be structured around the interests and background of the audience according to the following broad outline:

- Introduction to linear evolutionary equations in Banach spaces.
- Introduction to non-linear evolutionary equations in Banach spaces.
- Introduction to gradient inequalities and gradient flow equations.
- Application to one non-linear evolutionary equations selected in consultation with the students (Ginzburg-Landau equation, harmonic map heat flow, Ricci curvature flow, Yamabe curvature flow, or Yang-Mills gradient flow).

Students will be polled at the start of the class regarding their interests. The choice of topics and the level of coverage will be carefully adjusted for the audience at the beginning of the semester in order to accommodate different interests and backgrounds.

Prerequisites:

A graduate course on real analysis covering elementary measure theory (Lebesgue integral), and the concepts of Hilbert spaces and Banach spaces will be useful.

Co-requisites:

A one-semester graduate course on functional analysis (for example, based on the text by Haim Brezis) or partial differential equations (for example, based on the text by Lawrence Evans) would be useful, but will not necessarily be assumed. The course will be self-contained.
Grading:

This is an elective course for doctoral students. There are no formal course requirements (homework or exams) and the only requirement for a perfect grade is attendance. I will occasionally provide optional homework problems, but they will not be required.

Primary references:


Supplementary references:


Research articles: