

SUPPLEMENTARY QUESTIONS 2

Definition 1. For each subset $A \subseteq [0, 1]$, the corresponding two-player game G_A is defined as followed.

- Players I and II alternately pick decimal digits

$$a_1, b_1, a_2, b_2, \dots, a_n, b_n, \dots$$

with $0 \leq a_n, b_n \leq 9$.

- Player I wins if and only if

$$0 \cdot a_1 b_1 a_2 b_2 \dots a_n b_n \dots \in A.$$

Question 1. Prove that if A is a countable subset of $[0, 1]$, then Player II has a winning strategy in the game G_A .

Question 2. Prove that there exists a subset $A \subseteq [0, 1]$ such that neither Player I nor Player II has a winning strategy in the game G_A .