

## Some review problems, exam #1

Disclaimer: These are typical problems, but are not meant to be an exhaustive list. Your homework assignments and quizzes, together with these problems, are together meant to give you an idea of the kinds of problems that the exam will largely consist of. You are responsible for all the material in chapters 1 and 2, as well as some material from chapter 3: the objective row trick, section 3.1, and the dual simplex method.

1. Solve the following LPP by the 2-phase method: Minimize  $z = 2x_1 + x_2 + 2x_3 + x_4$  subject to

$$\begin{aligned}x_1 + x_3 + x_4 &\geq 10 \\x_2 + 2x_4 &\leq 12 \\2x_1 - 2x_2 - x_4 &= -10 \\x_1, x_2, x_3, x_4 &\geq 0\end{aligned}$$

In addition, state the equation for  $z$  which is implicit in the final tableau that you reach, and say for which  $\mathbf{x}$ 's this equation is valid.

2. (a) Solve the LPP: Maximize  $z = 17x_1 + 23x_2$  subject to

$$\begin{aligned}2x_1 + x_2 &\leq 16 \\x_1 + x_2 &\leq 10 \\x_1 + 2x_2 &\leq 16 \\2x_1 + 5x_2 &\leq 40 \\x_1, x_2 &\geq 0\end{aligned}$$

In addition,

- (b) sketch the feasible region in the  $x_1, x_2$ -plane (if there are any feasible solutions),
- (c) sketch the optimal level line of  $z$  (if there is one),
- (d) label the points corresponding to the tableaux,
- (e) determine which extreme points are degenerate (if any).

3. In the previous problem, suppose that after the LPP is solved, it is decided to change the problem by adding the constraint  $x_1 + 3x_2 \leq 20$ . By modifying the final tableau appropriately, obtain a tableau for the modified problem. What algorithm should be followed to get an optimal tableau? Carry it out.

4. Set up the initial tableau for Phase 1 of the two-phase method, for the following LPP: Maximize  $z = 493x_1 - 571x_2$  subject to the constraints  $x_1 + x_2 = 17$ ,  $x_2 + x_3 = 15$ ,  $2x_3 - x_1 \leq 0$ ,  $x_1 \geq 0$ ,  $x_2 \geq 0$ ,  $x_3$  unconstrained. Do not proceed further with the solution.

5. Consider the constraints

$$\begin{aligned}x_1 + x_3 + x_4 - x_5 + x_6 &= 10 \\x_2 + 2x_4 &= 12 \\2x_1 - 2x_2 - x_4 - 2x_5 &= -6 \\x_1, x_2, x_3, x_4, x_5, x_6 &\geq 0\end{aligned}$$

For each of the following vectors, determine if it is (a) a solution; (b) a basic solution; (c) a feasible solution; (d) a basic feasible solution; (e) a degenerate basic solution.

$$(1) \begin{bmatrix} 0 \\ 0 \\ 4 \\ 6 \\ 0 \\ 0 \end{bmatrix} \quad (2) \begin{bmatrix} 1 \\ 3 \\ -2 \\ 9 \\ 2 \\ 0 \end{bmatrix} \quad (3) \begin{bmatrix} 0 \\ 0 \\ -16/3 \\ 26/3 \\ -4/3 \\ 0 \end{bmatrix} \quad (4) \begin{bmatrix} 4 \\ 4 \\ 3 \\ 4 \\ 1 \\ 0 \end{bmatrix} \quad (5) \begin{bmatrix} 0 \\ 0 \\ 2 \\ 6 \\ 0 \\ 2 \end{bmatrix} \quad (6) \begin{bmatrix} 9 \\ 12 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

**6.** Know the definitions of all the relevant terminology, and the fundamental theorems behind the simplex method. This includes, but is not restricted to, the following: convex set; basic solution to a system  $A\mathbf{x} = \mathbf{b}$ ; what it means for a LPP to be in super-canonical form; extreme point of a subset  $S \subseteq \mathbf{R}^n$ ; the Extreme Point Theorem; the equivalence between BFS's of a LPP and extreme points of its feasible region.

**7.** Some multiple choice.

- (1) The reason that minimum  $\theta$  ratios are used to determine the departing variable is
  - a. to reach the optimal solution as fast as possible;
  - b. to have some rule – there is no other reason;
  - c. to make sure that the next tableau is feasible;
  - d. to avoid degeneracy;
  - e. none of the above.
- (2) The reason that the pivot entry is chosen positive and the corresponding entry of the objective row is chosen negative is
  - a. to reach the optimal solution as fast as possible;
  - b. to have some rule – there is no other reason;
  - c. to make sure that the next tableau is feasible;
  - d. to make sure that  $z$  is not strictly smaller in the next tableau;
  - e. none of the above.
- (3) The reason that the largest negative entry is used to determine the entering variable is
  - a. to reach the optimal solution as fast as possible;
  - b. to have some rule – any negative entry can be used;
  - c. to make sure that the next tableau is feasible;
  - d. to avoid degeneracy;
  - e. none of the above.
- (4) Cycling cannot occur in the simplex method
  - a. if Bland's Rule is used;
  - b. unless a degenerate basic feasible solution is encountered;
  - c. if the objective function strictly increases from tableau to tableau;
  - d. all of the above;
  - e. none of the above.
- (5) If four artificial variables have been introduced in a LPP, then in Phase 1,
  - a. at least four iterations will be required to reach a feasible solution;
  - b. if only three iterations are needed in Phase 1, then the original LPP is infeasible;
  - c. if an artificial variable remains in the basis at the end of Phase 1, then the original LPP is infeasible;
  - d. all of the above;
  - e. none of the above.