354-04: Quiz #3, 2/21/06 *SOLUTIONS*

1. Find the dual of the LPP

Minimize $z = 3x_1 - 4x_2 + 2x_3$ subject to

$$\begin{aligned} 5x_1 + 6x_2 - x_3 &\leq 12 \\ 7x_1 + 12x_2 &\geq 20 \\ x_1 \geq 0,\ x_2 \text{ unconstrained},\ x_3 &\geq 0 \end{aligned}$$

SOLUTION: Many equivalent answers are possible. Using the equivalence table from class, and because it is a minimization problem, we think of the LPP as being in "dual standard form" except that the first constraint has the inequality in the wrong direction, and the second variable is unconstrained. Therefore the dual will be in "standard form" except that the first variable will be ≤ 0 and the second constraint will be an equation:

Maximize $z_D = 12w_1 + 20w_2$ *subject to*

$$\begin{aligned} 5w_1 + 7w_2 &\leq 3 \\ 6w_1 + 12w_2 &= -4 \\ -w_1 &\leq 2 \\ w_1 \leq 0,\ w_2 &\geq 0 \end{aligned}$$

2. Suppose that while carrying out the dual simplex algorithm, you come to the following tableau. Circle the next pivot. Do not carry out the pivot.

	1	−2	3	−4	0	0	−10	5	−100
	0	−5	4	−3	0	1	12	−3	200
	0	−3	0	3	1	0	−2	4	−200
	0	1	1	1	0	0	1	1	

SOLUTION: The departing variable is given by the most negative entry in the resource column, so the pivot will be in the third row. The ratios to test are $|1/(-3)|$ *and* $|1/(-2)|$. *The first is the smaller, so: PIVOT ON THE* -3 *IN THE THIRD ROW.*