1. Write an equivalent LPP in standard form:

Minimize $z = -4x_1 + 2x_2 + x_3$, subject to

$$\begin{aligned} 3x_2 + 10x_3 &= 100 \\ 2x_1 + x_2 + x_3 &\geq 5 \\ x_2 \geq 0,\ x_3 &\geq 0 \end{aligned}$$

SOLUTION: Four things need fixing up. Instead of minimizing z, maximize $z' = -z$. Write $x_1 = x_1^+ - x_1^-$ since x_1 has no nonnegativity constraint. Replace $a = b$ by $a \leq b$ and $-a \leq -b$, and change the sign of the $\geq$ constraint:

Maximize $z' == 4x_1^+ - 4x_1^- - 2x_2 - x_3$, subject to

$$\begin{aligned} 3x_2 + 10x_3 &\leq 100 \\ -3x_2 - 10x_3 &\leq -100 \\ -2x_1^+ + 2x_1^- - x_2 - x_3 &\leq -5 \\ x_1^+ \geq 0,\ x_1^- \geq 0,\ x_2 \geq 0,\ x_3 &\geq 0 \end{aligned}$$

2. Explain why $\begin{bmatrix} 4 \\ 0 \\ 0 \\ -1 \end{bmatrix}$ is or is not a basic solution of the system $\begin{aligned} -x_1 - x_2 + 3x_3 + 3x_4 &= -7 \\ 2x_1 + x_2 - 3x_3 - 6x_4 &= 14 \end{aligned}$

SOLUTION: It is not basic. There are $m = 2$ equations, so a basic solution must have 2 basic variables. Since nonbasic variables are 0, the only choice is for x_1 and x_4 to be basic. So far, so good. But the columns $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ -6 \end{bmatrix}$ are not linearly independent (equivalently, the system $-x_1 + 3x_4 = -7,\ 2x_1 - 6x_4 = 14$ does not have a unique solution), and so the solution is not basic.