Some exercises from Section 2.6. Exercises 2 thru 5 will be done on the blackboard.

**Homomorphism theorems.** If \( \phi : G \to G' \) is a homomorphism, the image of \( G \), i.e.,

\[
G^* = \{ g' \in G' : (\exists g \in G) \phi(g) = g' \}
\]

= \{ \phi(g) : g \in G \}

is a subgroup of \( G' \), so \( \phi \) splits into a map onto \( G^* \) followed by the inclusion of \( G^* \) in \( G' \). Inclusions of subgroups seem almost too trivial to mention, so the mapping onto \( G^* \) gets most of the attention.

The kernel \( K \) of this mapping is the same as the kernel of \( \phi \) since the identity of the subgroup \( G^* \) is the identity element of \( G' \). The factor group construction leads to another group \( G/K \) with a homomorphism from \( G \) to \( G/K \) having \( K \) as kernel. The First homomorphism theorem says that \( G/K \) is isomorphic to \( G^* \) (and hence acts like a subgroup of \( G' \)) and this isomorphism is consistent with the homomorphisms we have from \( G \) to each of these groups.

The next result mentioned in this section is the Correspondence theorem that characterizes the sets

\[
H = \{ a \in G : \phi(a) \in H' \},
\]

where \( H' \) is a subgroup of \( G' \), as the subgroups of \( G \) that contain \( K \). Since \( K \) is a normal subgroup of \( H \), we can form \( K/H \). The theorem also relates \( H/K \) to \( H' \). If \( H' \) is a normal subgroup of \( G', H \) will also be a normal subgroup of \( G \).

The Second Homomorphism Theorem starts with an arbitrary subgroup \( H \) of \( G \) and considers its image in \( G/N \) for some \( N \triangleleft G \). The image of \( H \) is \( H/H \cap N \), and the correspondence theorem shows that this is isomorphic to the quotient of a group containing \( N \), which turns out to be \( HN \) by \( N \). This gives

\[
\frac{H}{H \cap N} \cong \frac{HN}{N}. \quad (\text{II})
\]

Finally, there is a Third Isomorphism Theorem that looks at the factor group of \( G' \) by a normal subgroup \( N' \) and relates it to \( G/N \) where

\[
N = \{ a \in G : \phi(a) \in N' \}.
\]
Proofs of these theorems, as well as exercises 2 and 6, will be done at the blackboard. This means that exercise 5 should be removed from the list of homework problems.

Section 2.8, along with its homework, will also be skipped at this time.