FINAL EXAM Math 251 Name:

PROBLEM 1 (5 pts): Find the equation of the plane containing the points $(1,0,-1),(1,1,2)$ and $(-1,2,9)$.

PROBLEM 2: The two space curves $\vec{r}_1(t) = \langle t+1,\ t^2-2t+2,\ 2t\rangle$ and $\vec{r}_2(s) = \langle 2s,\ s^3-2s+1,\ 5s-4\rangle$ intersect in exactly one point.
a (3 pts): Find the point of intersection and the values t_0 and s_0 that give this point for $\vec{r}_1$ and $\vec{r}_2$ respectively.
b (3 pts): Find the angle between the vectors $\vec{r}_1\,'(t_0)$ and $\vec{r}_2\,'(s_0)$.

PROBLEM 3 (12 pts): Find the points of maximum and minimum curvature for the curve $\vec{r}(t) = (e^t)\hat{\imath} + (t)\hat{\jmath}$ on the range $-2 \le t \le 2$.
PROBLEM 4 (12 pts): At the point $(-2,7)$ a certain function $f(x,y)$ has gradient $\nabla f(-2,7) = \langle -4, 12\rangle$. Find two unit vectors (directions) for which the directional derivative of f at $P = (-2,7)$ is equal to 4.

PROBLEM 5 (12 pts): Find and classify all critical points of the function

$$f(x,y) = xy^2 - 6x^2 - 3y^2 + 10.$$

PROBLEM 6 (16 pts): Use the method of Lagrange Multipliers to find the points on the "squircle" $x^4 + y^4 = 1$ that are closest to and furthest from the origin.
PROBLEM 7 (12 pts): Find the volume of the solid bounded below by the cone $z = \sqrt{x^2+y^2}$ and bounded above by the circular paraboloid $z = 2 - (x^2+y^2)$.
PROBLEM 8: Consider the transformation

$$T \ :\ \begin{cases} x = u^2 - v^2 \\ y = uv \end{cases}$$

a (6 pts): Give a sketch of the image in the xy-plane of the (u,v) rectangle $-1 \le u \le 1$ and $0 \le v \le 1$ under this transformation. Indicate equations for the boundary curves.
b (6 pts): What is the Jacobian of this transformation?
PROBLEM 9 (12 pts): Compute the integral of the function $f(x,y,z) = x^2+y^2+z^2$ over the curve $\vec{r}(t) = (4\cos(t))\hat{\imath} + (3t)\hat{\jmath} + (4\sin(t))\hat{k}$ for $0 \le t \le 2$.

PROBLEM 10 (16 pts): Compute the integral of the vector field $\vec{F}(x,y) = \langle 2xy^2+1, 2x^2y+2y+1\rangle$ over the arc of the circle $(x+1)^2+(y+\frac{1}{2})^2 = \frac{13}{4}$ starting at $(0,1)$ and ending at $(-2,-2)$

PROBLEM 11 (12 pts): Is the vector field $\vec{F}$ conservative?

$$\vec{F}(x,y,z) = (e^{y^2+z}+yz)\hat{\imath} + (2xye^{y^2+z}+xz)\hat{\jmath} + (xe^{y^2+z}+xy+2z)\hat{k}$$

If so, find a potential function $f(x,y,z)$ so that $\nabla f(x,y,z) = \vec{F}(x,y,z)$.

PROBLEM 12 (16 pts): Compute the integral of the vector field

$$\vec{F}(x,y) = (-3xy^2+3x^2y+\sqrt{x^2+1})\hat{\imath} + (3x^2y+x^3+e^{\cos(y)})\hat{\jmath}$$

over the curve starting at the origin and moving to $(1,0)$ along the x-axis, then moving vertically from $(1,0)$ to $(1,9)$ and finally moving along the curve $y = 10x - x^2$ from $(1,9)$ back to the origin.

PROBLEM 13 (12 pts): Compute $\int\int_{\mathcal{S}} y\, dS$ over the triangular region in 3-space with vertices $(4,0,0), (0,4,0)$ and $(0,0,2)$.

PROBLEM 14 (16 pts): Use Stokes' Theorem to calculate $\oint_C \vec{F}\cdot d\vec{r}$ where $\vec{F}(x,y,z) = \langle 2z, x, 3y\rangle$ and C is the ellipse that is the intersection of the cylinder $x^2+y^2=4$ and the plane $z=x$ oriented counterclockwise when viewed from above.

PROBLEM 15 (16 pts): Use the Divergence Theorem to calculate the flux $\int\int_{\mathcal{S}} \vec{F}\cdot d\vec{S}$ where

$$\vec{F}(x,y,z) = x^2\hat{\imath} + y^2\hat{\jmath} + z^2\hat{k}$$

and $\mathcal{S}$ is the parabolic solid given by $0 \le z \le 4-(x^2+y^2)$.

PROBLEM 16 (12 pts): Let $\mathcal{S}$ be the open hemisphere of $x^2+y^2+z^2=16$ where $z \ge 0$ oriented upwards (by open, we mean that the bottom of the hemisphere IS NOT INCLUDED). Compute $\int\int_{\mathcal{S}} \vec{F}\cdot d\vec{S}$, where

$$\vec{F}(x,y,z) = (xy^2+e^{yz})\hat{\imath} + (yx^2+\cosh(x^2+2z))\hat{\jmath} + (\frac{1}{3}z^3+x^2+y^2)\hat{k}.$$