640:250:C3 Spring 2001

ADDITIONAL REVIEW PROBLEMS—ANSWERS

1. $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = C_1 e^{-t} \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + C_2 e^{-t} \begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} + C_3 e^{-2t} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$.

Equivalently $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = C_1 e^{-t} \begin{bmatrix} 4/3 \\ 1 \\ 0 \end{bmatrix} + C_2 e^{-t} \begin{bmatrix} 1/3 \\ 0 \\ 1 \end{bmatrix} + C_3 e^{-2t} \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$.

2. $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Ellipse.

3. (a) Eigenvalues: $\lambda = -1 \pm \sqrt{1+2t}$, and for each of these values of λ the eigenvectors are $y \begin{bmatrix} \lambda/2 \\ 1 \end{bmatrix}$, $y \neq 0$. (b) $t = t_0 = -1/2$ is unique. (c) They all approach scalar multiples of the same vector $\begin{bmatrix} -1/2 \\ 1 \end{bmatrix}$.

4. (a) F, but true if A is also assumed real symmetric positive definite.

Counterexamples: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$.

(b) F, but the sum of the diagonal entries must be positive.

Counterexample: $\begin{bmatrix} 4 & 6 \\ -1 & -1 \end{bmatrix}$

(c) T (d) T

(e) F. If A is not invertible, then 0 is an eigenvalue.

Counterexample: $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is diagonalizable but not invertible.

5. $U = \begin{bmatrix} -\sqrt{1/3} & \sqrt{2/3} \\ \sqrt{2/3} & \sqrt{1/3} \end{bmatrix}$, $\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and $V = \begin{bmatrix} -\sqrt{2/3} & \sqrt{1/3} \\ \sqrt{1/3} & \sqrt{2/3} \end{bmatrix}$.

6. $\begin{bmatrix} 4/11 & 4/11 \\ 7/11 & 7/11 \end{bmatrix}$.

(Note: $A = S\Lambda S^{-1}$ where $S = \begin{bmatrix} 4 & 1 \\ 7 & -1 \end{bmatrix}$ and $\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & -0.1 \end{bmatrix}$.)

7. $\begin{bmatrix} -1 & 6 \\ -3 & 5 \end{bmatrix} = S\Lambda S^{-1}$ where $\Lambda = \begin{bmatrix} 2+3i & 0 \\ 0 & 2-3i \end{bmatrix}$ and $S = \begin{bmatrix} 1-i & 1+i \\ 1 & 1 \end{bmatrix}$.