

1.  $N(A)$ :  $(1, -2, 1)$ , dimension 1.  
 $C(A)$ : first two columns of  $A$ , dimension 2.  
 $R(A)$ :  $(1, 1, 1)$  and  $(1, 2, 3)$ , or  $(1, 1, 1)$  and  $(0, 1, 2)$  (lots of possibilities), dimension 2.  
 $N(A^T)$ :  $(2, -1, 1, 0)$  and  $(3, -5, 0, 1)$ ; dimension 2.
2. (a)  $-54$  (b)  $\det A \neq 0$   
 (c) (from cofactor formula for inverse) the fourth row of  $A^{-1}$  is  $[0 \ 0 \ 1/3 \ 0 \ 0]$ .
3. (a) The determinant is divided by 8. (b) The determinant is unchanged. (c) The determinant is tripled. (d) The determinant is unchanged.
4. (a) does not span, not LI (b) does not span but is LI (c) spans and is LI
5. The first two form a basis; the space has dimension 2.  
 A basis of  $\mathbf{R}^4$  is  $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$  (many answers are possible).
6. (a) T (b) T (c) F (to correct it remove “no two of which are scalar multiples of each other” and replace it with “none of which is a linear combination of the others”) (d) T (e) T
7.  $N(A)$ : 102 and 200.  $C(A)$ : 98 and 100.  $R(A)$  or  $C(A^T)$ : 98 and 200.  $N(A^T)$ : 2 and 100.
8. (a)  $m = n$ . (b) The pivot columns are the first  $n$  columns. (c) The first  $n$  rows are all nonzero and the remaining rows are all  $\mathbf{0}$ .
9.  $-13/18$ .
10.  $y = 1.9x + 3.75$
11. (a)  $(-1/3, 0, -1/6, 1/6)$  (b)  $(-0.2, 0.4, 0.1, 0.7)$ .
12.  $Q = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 1/2 & -1/2 & -1/\sqrt{2} \end{bmatrix}$  and  $R = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$ .
13.  $\lambda_1 = 1$ , with eigenvectors:  $t \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ ; and  
 $\lambda_2 = 8$ , with eigenvectors:  $t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ .