1. (a) $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -4 & -1 & 1 \end{bmatrix}$, $U = \begin{bmatrix} 1 & -1 & 3 \\ 0 & -2 & -4 \\ 0 & 0 & -3 \end{bmatrix}$.

(b) $E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$, $E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

(c) $L = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1}$.

(d) $L =$ same as (a), $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$, $U = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$.

(e) (see (d)) $L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$, $U = \begin{bmatrix} 1 & -2 & 4 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(i) $\mathbf{c} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 11/2 \\ 3/2 \\ -1 \end{bmatrix}$.

(k) $C(A) = \mathbf{R}^3$, $N(A) = \{\mathbf{0}\}$.

(l,m) Impossible.

2. (a) $a = 0$, $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $L = I$, $U = \begin{bmatrix} 2 & -6 \\ 0 & 3 \end{bmatrix}$.

(b) $a = -1$, $\mathbf{b} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ or any scalar multiple, and then $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ or any vector not a scalar multiple of $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

(c) $\mathbf{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ or any nonzero scalar multiple of it; and $\mathbf{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ or any nonzero scalar multiple of it.

3. (c) It's $C(A)$.

4. $L = U = I$; $L = D = U = I$.

5. (a) Pivot: 2 and 4. Free: 1, 3, 5 and 6. Rank is 2, nullity 4.

(b) $(1,0,0,0,0,0), (0,-3,1,0,0,0), (0,4/3,0,4/3,1,0), (0,-5/3,0,4/3,0,1)$

(c) Columns 2 and 4 of A.

6. FTFT

7. (c) The two vectors have their tips on the line, but their sum does not.

8. (a) Yes, $\mathbf{v} \cdot \mathbf{w} = 0$.

9. (aj)(bcdgi)(eh)(fk)