Jan. 18, 2001 QUIZ, MATH 250–C3 Name: _____

Signature: _____

**1.** In each case, mark with S, V or N to indicate that the entire expression represents a scalar, represents a vector, or does not make sense. Assume that $\mathbf{v}$, $\mathbf{w}$ and $\mathbf{x}$ are real column vectors of the same size.

a) $\mathbf{v} \bullet (\|\mathbf{w}\|\mathbf{x})$

**Solution:** S. $\|\mathbf{w}\|$ is a scalar, so $(\|\mathbf{w}\|\mathbf{x})$ is a vector, and the dot product of two vectors is a scalar.

b) $\mathbf{v} \bullet (\mathbf{w} \bullet \mathbf{x})$

**Solution:** N. $(\mathbf{w} \bullet \mathbf{x})$ is a scalar and the dot product of a vector with a scalar is undefined.

c) $\dfrac{\mathbf{v} \bullet \mathbf{w}}{\mathbf{w} \bullet \mathbf{w}}\mathbf{v}$ (assuming that $\mathbf{w} \neq \mathbf{0}$)

**Solution:** V. Since $\mathbf{w} \neq \mathbf{0}$ and $\mathbf{w}$ is a real vector, $\mathbf{w} \bullet \mathbf{w} = \|\mathbf{w}\|^2 \neq 0$. Both $\mathbf{v} \bullet \mathbf{w}$ and $\mathbf{w} \bullet \mathbf{w}$ are scalars, so the fraction is a scalar, and multiplying the vector $\mathbf{v}$ by that scalar yields a vector.

**2.** Express $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 5 \end{bmatrix}$.

**Solution:** $\begin{bmatrix} 3 \\ 2 \end{bmatrix} = x \begin{bmatrix} 5 \\ 4 \end{bmatrix} + y \begin{bmatrix} 7 \\ 5 \end{bmatrix}$ leads to

$$\begin{aligned} 5x + 7y &= 3 \\ 4x + 5y &= 2 \end{aligned}$$

which can be solved in many ways to give $x = -1/3,\ y = 2/3$. So

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 5 \\ 4 \end{bmatrix} + \frac{2}{3}\begin{bmatrix} 7 \\ 5 \end{bmatrix}.$$