Mar. 27, 2001 QUIZ, MATH 250–C3 Name: ______________________

Signature: ______________________

1. Gaussian elimination reduces a certain matrix A to $\begin{bmatrix} -2 & 3 & 9 \\ 0 & 1 & 4 \\ 0 & 0 & 4 \end{bmatrix}$.

a) If no row interchanges were involved in the elimination, what would $\det A$ have to be?

Solution: $\det A = (\text{product of pivots}) \cdot (-1)^{\text{number of row interchanges}} = (-2)(1)(4)(-1)^0 = -8$.

b) If exactly three row interchanges were involved in the elimination, what would $\det A$ have to be?

Solution: $\det A = (-2)(1)(4)(-1)^3 = 8$.

2. True or false for every square matrix A? Correct the statements which are false.

a) $\det(-A) = -\det A$.

Solution: False in general. If A is an $n \times n$ matrix then $\det(cA) = c^n \det(A)$. So $\det(-A) = (-1)^n \det A$. The statement is true, however, for $n \times n$ matrices with n even.

b) If the rows of A form a LI set, then $\det A = 0$.

Solution: True. If some row is 0 then $\det A = 0$. If some row (say the ith row) is a linear combination of the others, then by subtracting multiples of the other rows from the ith row, we do not change the determinant, but reach a matrix B with a zero row. So $\det(A) = \det(B) = 0$.

c) If the rows of A form a LI set, then so do the columns of A.

Solution: True. Suppose that A is $n \times n$. Then:

$$\begin{aligned} \text{The rows of } A \text{ form a LI set } &\Longrightarrow \\ \text{the rows of } A \text{ form a basis of } R(A) &\Longrightarrow \\ \dim(R(A)) = n &\Longrightarrow \\ \dim(C(A)) = n &\Longrightarrow \text{the columns of } A \text{ (which span } C(A)\text{, after all) are LI.} \end{aligned}$$