Math 250-C1 **Linear Algebra, SOLUTIONS, QUIZ 9**

Let $A = \begin{bmatrix} -9 & 8 & -8 \\ -4 & 3 & -4 \\ 2 & -2 & 1 \end{bmatrix}$

The characteristic polynomial of A is $-(t+3)(t+1)^2$. Find, if possible, an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. Otherwise, explain why A is not diagonalizable.

The eigenvalues of A are $t = -3$ and $t = -1$.

The set of eigenvectors of A associated to eigenvalue -1 is the set of non-zero vectors in null space of

$$A + I = \begin{bmatrix} -8 & 8 & -8 \\ -4 & 4 & -4 \\ 2 & -2 & 2 \end{bmatrix}.$$

This matrix row reduces to $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. The nullity is two and $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis, so it is a set of linearly independent eigenvalues spanning the eigenspace for -1.

Similarly

$$A + 3I = \begin{bmatrix} -6 & 8 & -8 \\ -4 & 6 & -4 \\ 2 & -2 & 4 \end{bmatrix}.$$

It reduced row echelon form is $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$. Its null space, the eigenspace for eigenvalue -3, is spanned by $\begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}$.

We see that A has three linearly independent eigenvectors, which, since there are three, span $\mathcal{R}^3$. Thus A is diagonalizable, and $A = PDP^{-1}$, where

$$P = \begin{bmatrix} -4 & 1 & 0 \\ -2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$