Math 250-C1 **Quiz 6 solutions**

(a) Let $U = \begin{bmatrix} 2 & 1 & -3 \\ 0 & 3 & 6 \\ 0 & 0 & 4 \end{bmatrix}$. Find $\det(U)$. Explain how you get your answer.

B is upper triangular. Hence its determinant is the product of its diagonal entries. As a result, $\det(B) = 24$.

(b) A is a 3×3 matrix. The successive row operations $R1 \leftrightarrow R3$, $R_2 \leftarrow R_2 - 5R_1$, $R_3 \leftarrow R_3 + 4R_2$ transform A into the matrix U of part **(a)**. Find $\det(A)$. Explain how you get your answer.

The row operation of interchanging the first and third rows ($R1 \leftrightarrow R3$) produces from A a new matrix A_1. The effect is to change the sign of the determinant: $\det(A_1) = -\det(A)$.

The operation $R_2 \leftarrow R_2 - 5R_1$ produces from A_1 the matrix A_2 and the operation $R_3 \leftarrow R_3 + 4R_2$ produces from A_2 the matrix U, but neither of these operations changes the value of the determinant; thus $\det(U) = \det(A_2) = \det(A_1) = -\det(A)$. From the answer to part (a), $\det(A) = -24$.

(c) Let $B = \begin{bmatrix} 0 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & -5 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$. Calculate $\det(B)$ *without* multiplying the three matrices. Explain how you get your answer.

$$\det(B) = \left(\det \begin{bmatrix} 0 & 2 \\ 4 & 0 \end{bmatrix}\right) \left(\det \begin{bmatrix} 1 & -5 \\ 0 & 2 \end{bmatrix}\right) \left(\det \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\right) = (-8)(2)(1) = -16.$$